

# Classification of Edge-dependent Labels of Nodes in Hypergraphs

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Code and Data: <https://github.com/young917/EdgeDependentNodeLabel>

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## Summary

- Novel Problem:** Classification of edge-dependent node labels
- Effective Model:** A novel hypergraph neural network WHATsNET based on two ideas:
  - WithinATT:** An attention module where the edge-dependent embedding of each node is computed by attending to the other nodes within each hyperedge
  - WithinOrderPE:** An edge-dependent positional encoding defined by the relative order of node centrality within each hyperedge
- Extensive Experiments:**
  - Superiority:** WHATsNET performs significantly and consistently better than ten competitors on six real-world hypergraphs
  - Usefulness:** WHATsNET is demonstrated useful in three applications: (a) ranking aggregation, (b) node clustering, and (c) product return prediction

## Problem Definition: Edge-Dependent Node Classification

- Examples of Edge-Dependent Node Classification:**

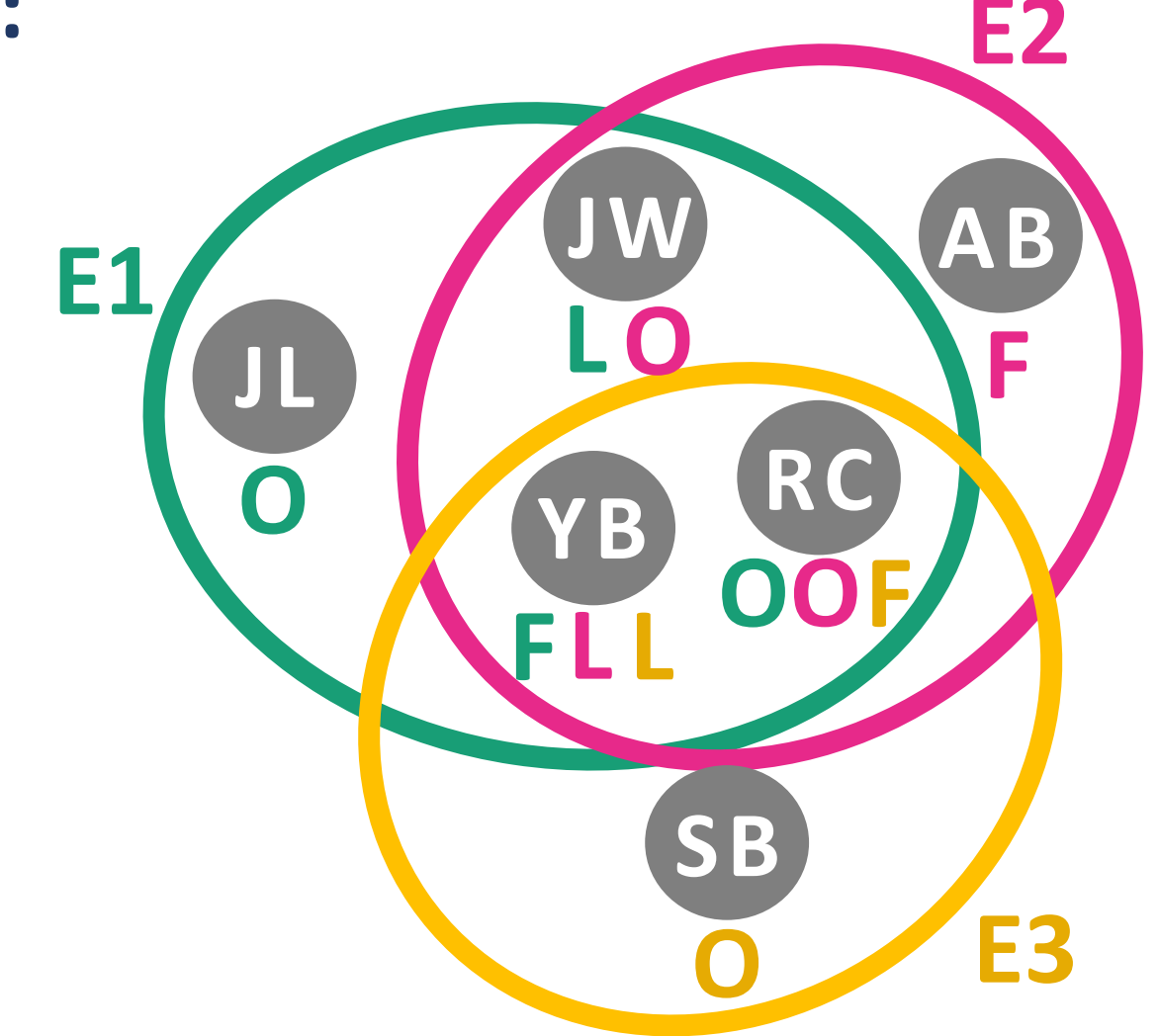
### (a) Co-Authorship

#### Authors (Nodes)

Y Bengio (YB)	J Weston (JW)
J Louradour (JL)	A Bordes (AB)
R Collobert (RC)	S Bengio (SB)

#### Publications (Hyperedges)

<b>E1:</b> Curriculum learning Y Bengio, J Louradour, R Collobert, J Weston – ICML'09
<b>E2:</b> Learning structured embeddings of knowledge bases A Bordes, J Weston, R Collobert, Y Bengio – AAAI'11
<b>E3:</b> A parallel mixture of SVMs for very large scale R Collobert, S Bengio, Y Bengio – NIPS'01



**Labels: First, Last, or Others** based on the positions in the author list of each publication

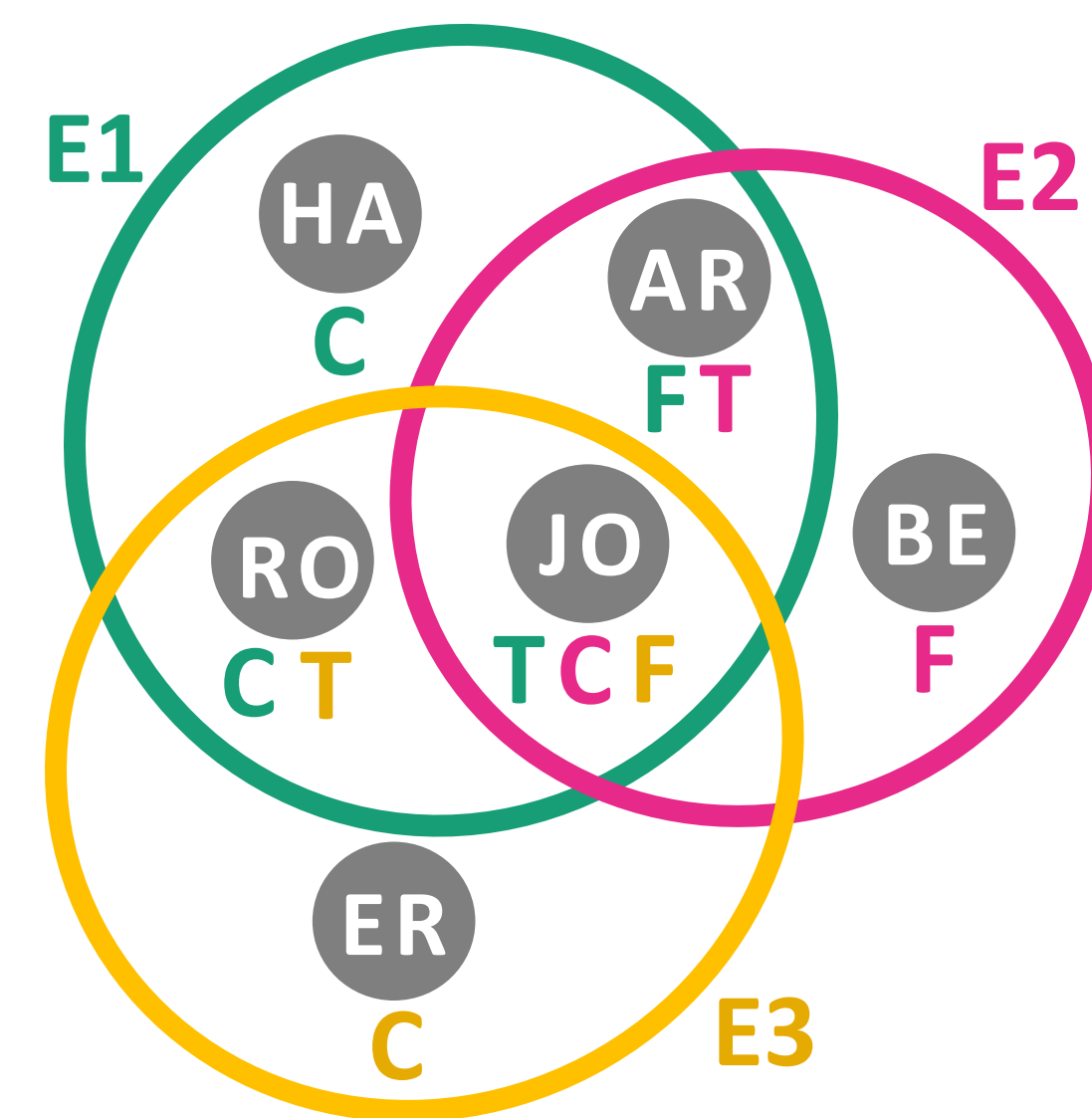
### (b) Email

#### People (Nodes)

Joan@enron.com (JO)	arnold@enron.com (AR)
harry@enron.com (HA)	ben@enron.com (BE)
robert@enron.com (RO)	eric@enron.com (ER)

#### Emails (Hyperedges)

<b>Email</b>	<b>Email</b>
To: Joan@enron.com	To: arnold@enron.com
CC: harry@enron.com, robert@enron.com	CC: Joan@enron.com
From: arnold@enron.com	From: ben@enron.com
TITLE: Today's meeting	TITLE: Project Issue
<b>Email</b>	<b>Email</b>
To: robert@enron.com	To: eric@enron.com
CC: eric@enron.com	From: joan@enron.com
From: joan@enron.com	TITLE: Discussion



**Labels: To, From, or CCs** based on the roles (senders, receivers, and the CCed) in each email

### Formal Problem Definition:

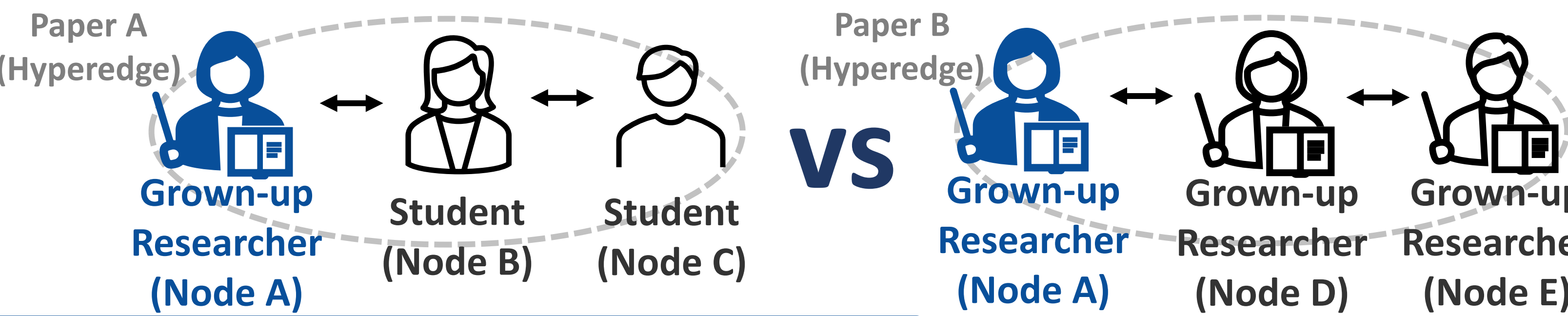
- Given:** (a) A hypergraph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ : a node set  $\mathcal{V}$  and a hyperedge set  $\mathcal{E}$
- (b) Edge-dependent node labels in some hyperedges  $\mathcal{E}' \subset \mathcal{E}$ :  $y_{v,e}, \forall v \in e, \forall e \in \mathcal{E}'$
- (c) (Optionally) a node feature matrix  $X$
- Aim to:** accurately predict the unknown edge-dependent node labels in  $\mathcal{E} \setminus \mathcal{E}'$ :  $y_{v,e}, \forall v \in e, \forall e \in \mathcal{E} \setminus \mathcal{E}'$

### Desirable Properties as a Benchmark Task for Hypergraph Neural Networks

- It evaluates the capability of capturing properties unique to hypergraphs
- Existing hypergraph neural networks exhibit limited performance in this task
- The predictive outputs can directly be applied to various applications

## WithinATT: Attention to Other Nodes Within Hyperedges

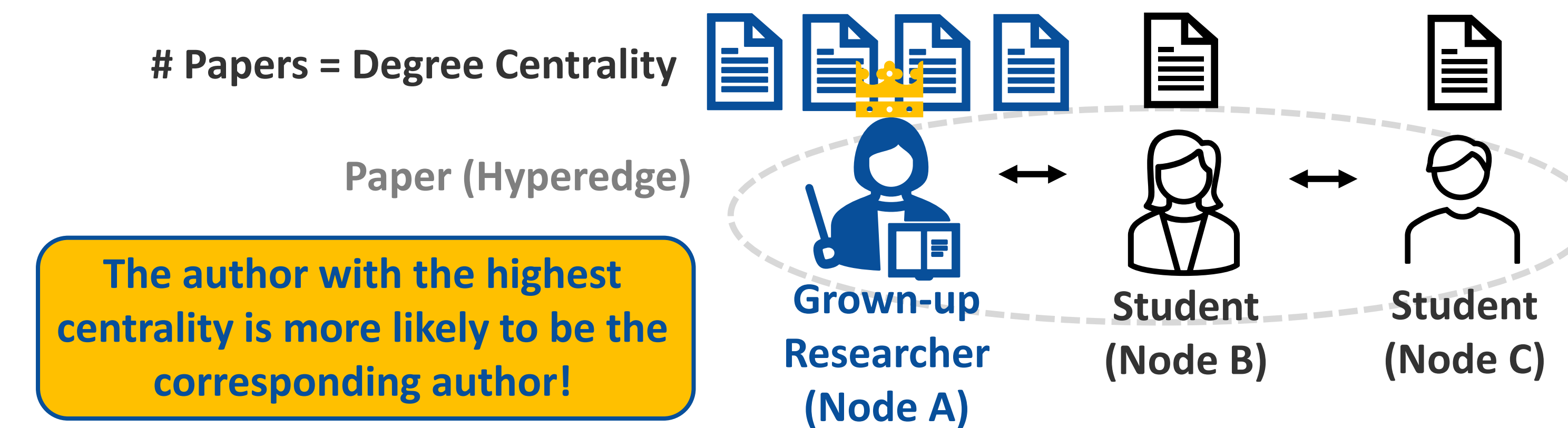
- Motivation:** The edge-dependent label of each node is shaped by its relation to the other nodes within the same hyperedge.
- Details:** WithinATT computes the **edge-dependent embedding** of each node by **attending** to the other nodes within each hyperedge



more likely to be the corresponding author in this case

## WithinOrderPE: Using Centrality for Positional Encoding

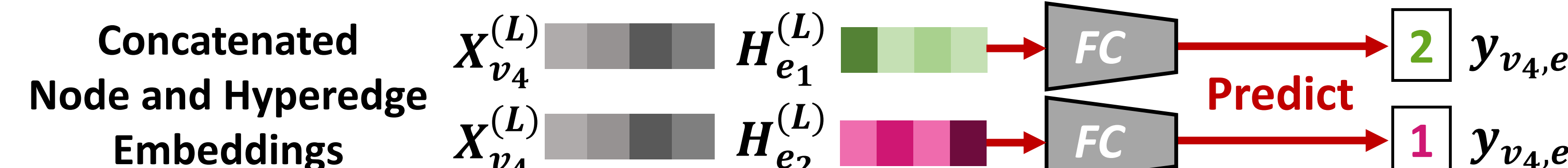
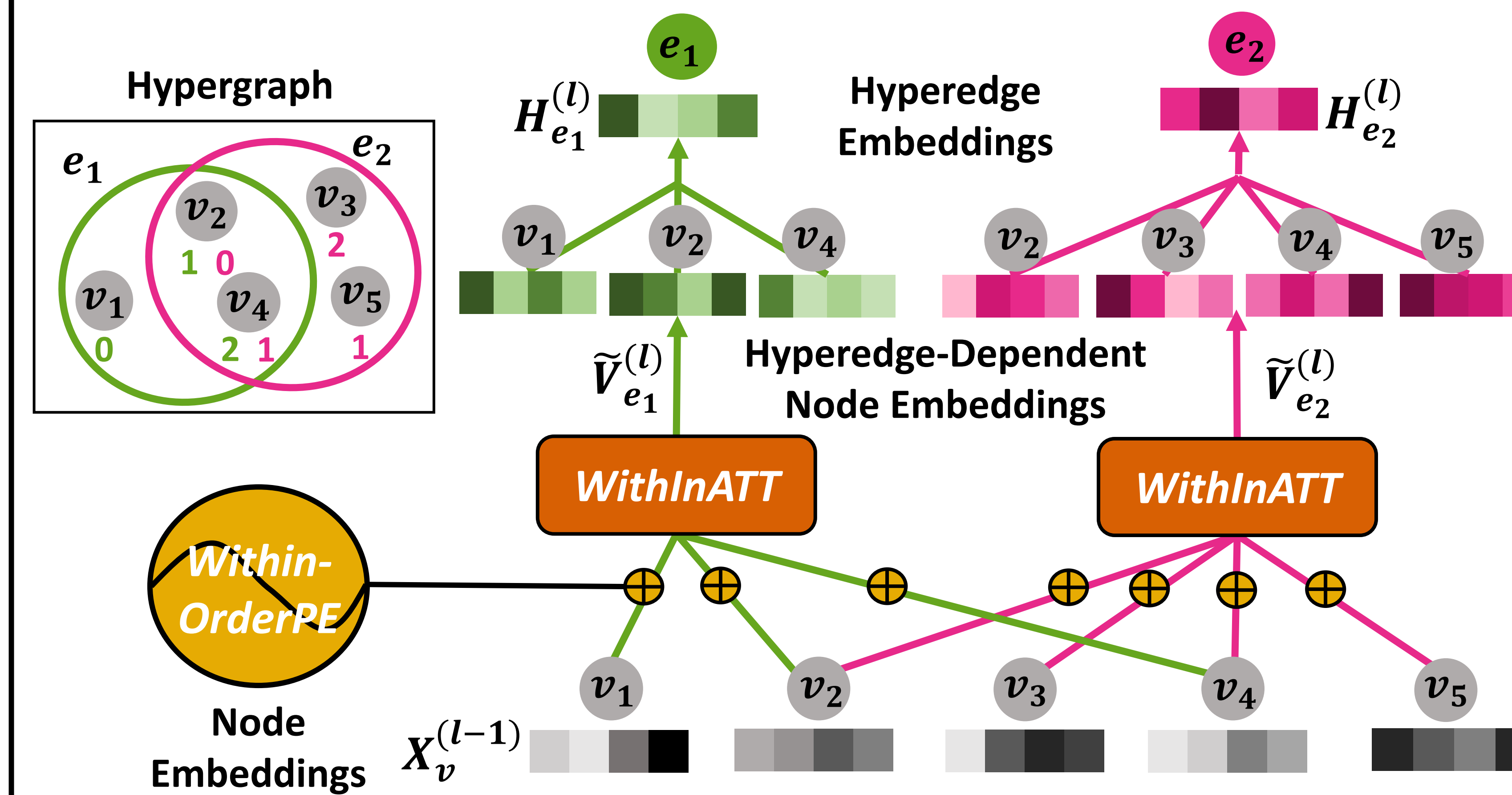
- Motivation:** The edge-dependent label of each node is closely related to its relative centrality (e.g., degree centrality) within each hyperedge
- Details:** WithinOrderPE provides the **edge-dependent positional encoding** of each node based on its centrality order within each hyperedge



The author with the highest centrality is more likely to be the corresponding author!

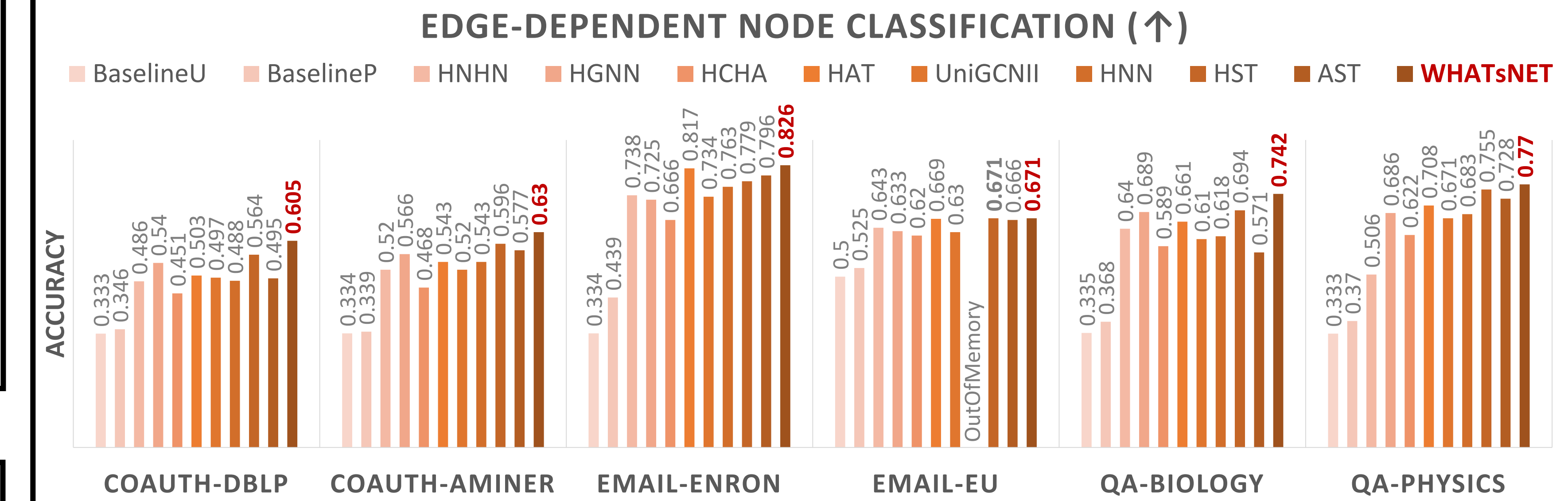
## Proposed Model: WHATsNET (Within-Hyperedge Attention Transformer Network)

- Two message passing steps in each layer:**
  - (1) Updating hyperedge embeddings by aggregating embeddings of incident nodes
  - (2) Updating node embeddings by aggregating embeddings of incident hyperedges
- In each step:**
  - (1) Input embeddings of nodes (or hyperedges) are augmented by WithinOrderPE
  - (2) WithinATT generates hyperedge-dependent node (or node-dependent hyperedge) embeddings
  - (3) They are aggregated into hyperedge (or node) embeddings
- Predict edge-dependent node labels:**
  - A single-layer perceptron classifier takes the concatenated node and hyperedge embeddings from the last layer of WHATsNET and produce estimated labels

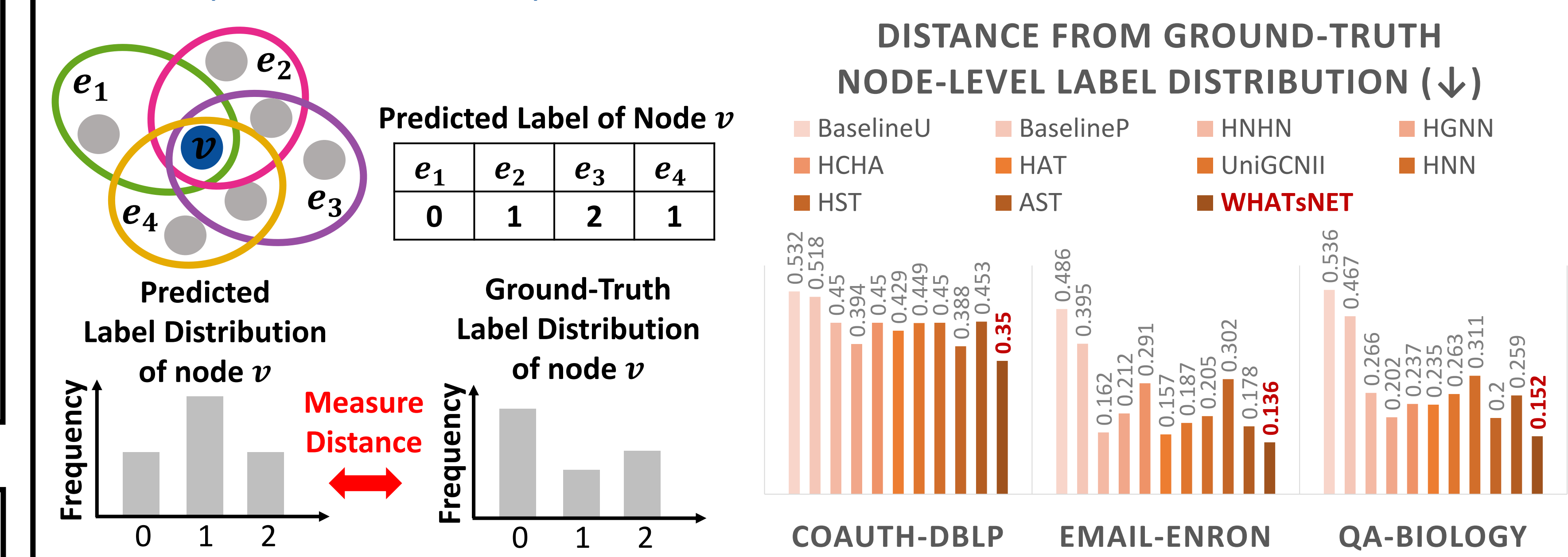


## Experimental Results

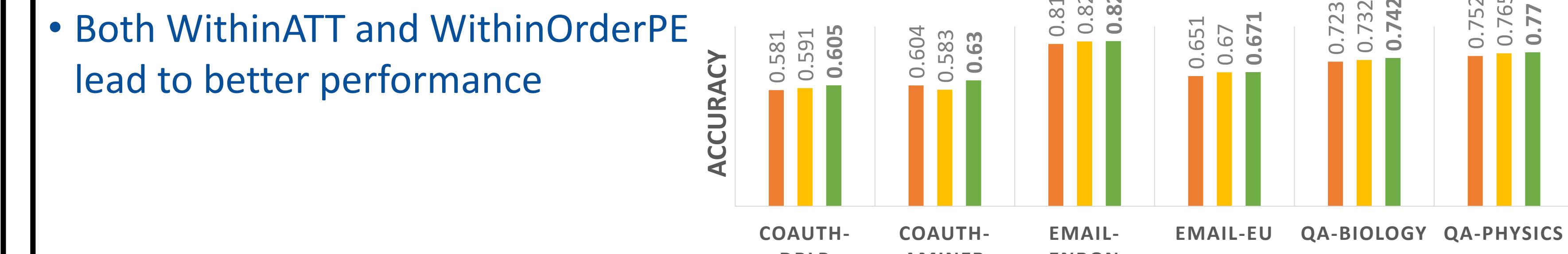
- Q1:** Does WHATsNet accurately predict the edge-dependent labels of nodes?
- WHATsNET consistently outperforms 10 competitors in classifying edge-dependent node labels across all 6 datasets



- Q2:** Does WHATsNet classify the same node differently depending on hyperedges?
- We investigate the distribution of edge-dependent labels of each node
- The outputs of WHATsNet preserve well the distribution



- Q3:** Does each component contribute to the performance?
- Both WithinATT and WithinOrderPE lead to better performance



- Q4:** Is WHATsNet useful in downstream tasks?

- Edge-dependent labels produced by WHATsNet consistently enhance the performance in three downstream tasks, compared to using no labels

