How Transitive Are Real-World Group Interactions? – Measurement and Reproduction





• Goal: To analyze transitivity of real-world group interactions and to develop a realistic and scalable synthetic hypergraph generator.

• Previous Work: Focus on transitivity of pairwise interactions but not on group interactions. • Contribution 1) Axioms: Criteria that any well-defined transitivity measures should meet. • Contribution 2) Principled Measure: A measure (HyperTrans) that meets all the criteria. • Contribution 3) Observations: Transitivity patterns of real-world hypergraphs. • Contribution 4) Generator: A scalable hypergraph generator (THera) that can reproduce

the transitivity patterns of real-world hypergraphs.

Reproduction: More accurately reproduce transitivity patterns than competitors. **Scalability:** Can generate hypergraphs of $10^{7.5}$ hyperedges within a minute. (5)

Observation

- Q3) What are the transitivity-related patterns of real-world group interactions?
- A3) Observations: We analyze transitivity of 12 real-world hypergraphs.
- Email & Drug & Student Communication & Co-Authorship & QnA
- Stack**Exchange**

Microsoft

FDA

- Null Model: To verify that the patterns are not random, we generate random counterparts of real-world hypergraphs by using HyperCL [2].
- **Observation 1:** Real-world Hypergraphs are transitive.



Measured hypergraph	transitivity values	(x100)
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Data E1 E2 D1 D2 S1 S2 C1 C2 C3 Q1 Q2 Q3	-							~ -				• -	
	Data	E1	E2	D1	D2	S1	S2	C1	C2	C3	Q1	Q2	Q3

** Oral Presentation Info: Tuesday, August 8, 10:00 AM - 12:00 AM, Room 201B.

Motivation

• Hypergraph: Group interactions are prevalent around us, and a hypergraph is a data structure that is widely used to represent such group interactions.







• Graph Transitivity: Probability of two neighbors of a node being also adjacent [1].





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0. 0.5 | 0.3 Null 7.8 5.3 0.8 0.5 11.9 22.3 0.0* 0.0* 0.0* 1.4 1.7 Real Random 0.0^* : Value $\leq 10^{-4}$ • Observation 2: Interactions with larger intersection tend to be more transitive. *Correlation between intersection and transitivity (x100).* Data E1 E2 D1 S1 S2 C2 C3 Q1 Q2 Q3 32 12 Real 14 13 14 12 13 12 0.0* 0.0* 0.0* 0.0* 0.0* 0.0* -9 -14 -1 -1 • **Observation 3:** High degree nodes tend to be less transitive. coauth-history gna-math Real-world Node nsitiv Random 10^{1} Extroverted Introverted Node Degree • Observation 4: Real-world hyperedges have a wide transitivity range. Range of hyperedge transitivity (x100). \circ \circ \circ \circ Real . . . E2 E1 | D2 S1 S2 D1 C2 Data C3 Q1 Q2 C1 Q3 Random **Real** 72.5 80.9 60 100 | 79.4 | 69.3 | 100 | 100 | 100 | 66.7 | 66.7 | 66.7 Null 27.9 24.8 7.5 3.2 31.6 39.5 10.5 6.9 33.3 50 33.3 100 \mathcal{T} : 1 ······ 0.2 Link Prediction ---- **O**

Generator

- Research Interest: Transitivity of real-world group interactions. +
 - No existing studies focus on the transitivity of group interactions.
 - Q1) How should we measure transitivity of group interactions?
 - Q2) How transitive are real-world group interactions?
 - Q3) How can we reproduce realistic transitivity patterns?

Axiom & Measure

• Q1-1) What are the criteria that any well-defined transitivity measure should meet? • A1-1) Axioms: We propose 8 axioms that formalize characteristics that a well-defined transitivity measure (\mathcal{T}) should satisfy.



Axioms: Criteria of a good measure



Axiom 1: Conditions for **Axiom 2:** Conditions for the minimum transitivity the increase in transitivity

• Q1-2) How can we systematically quantify the transitivity of group interactions? • A1-2) HyperTrans: Our proposed principled transitivity measure that satisfies all the

- Q4) How can we reproduce transitivity patterns of real-world hypergraphs? • A4) THera: We propose a scalable hypergraph generator that can reproduce transitivity patterns of real-world hypergraphs. • Description: THera mainly utilizes (1) community and (2) hierarchical structure of nodes. It stochastically creates two types of hyperedges:
 - Intra-community: THera assigns a community to each node and create hyperedges within each community.
 - Hierarchical: THera creates tree of nodes and sample nodes proportional to the level of each node.





Intra-community

Hierarchical

• Evaluation: THera outperforms all competitors in terms of (1) pattern reproducibility,

Generators	H-LAP+ [2]	ше Ш
Pattern reproducibility 1.1 4.23.93.0	2.8	Inti



proposed axioms (No baseline measures satisfy all the proposed axioms!).

