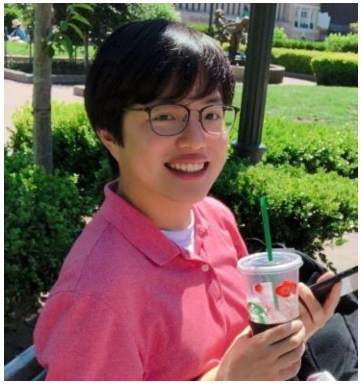
KAIST

SIAM 2022 International Conference on Data Mining

# On the Persistence of Higher-Order Interactions in Real-World Hypergraphs



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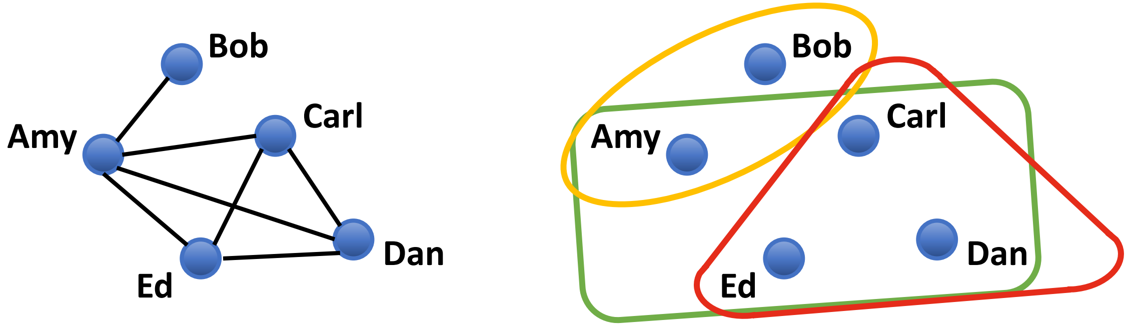
Kijung Shin

# Hypergraph

- A **hypergraph** is a generalization of an ordinary graph
- A **hyperedge** joins an **arbitrary** number of nodes

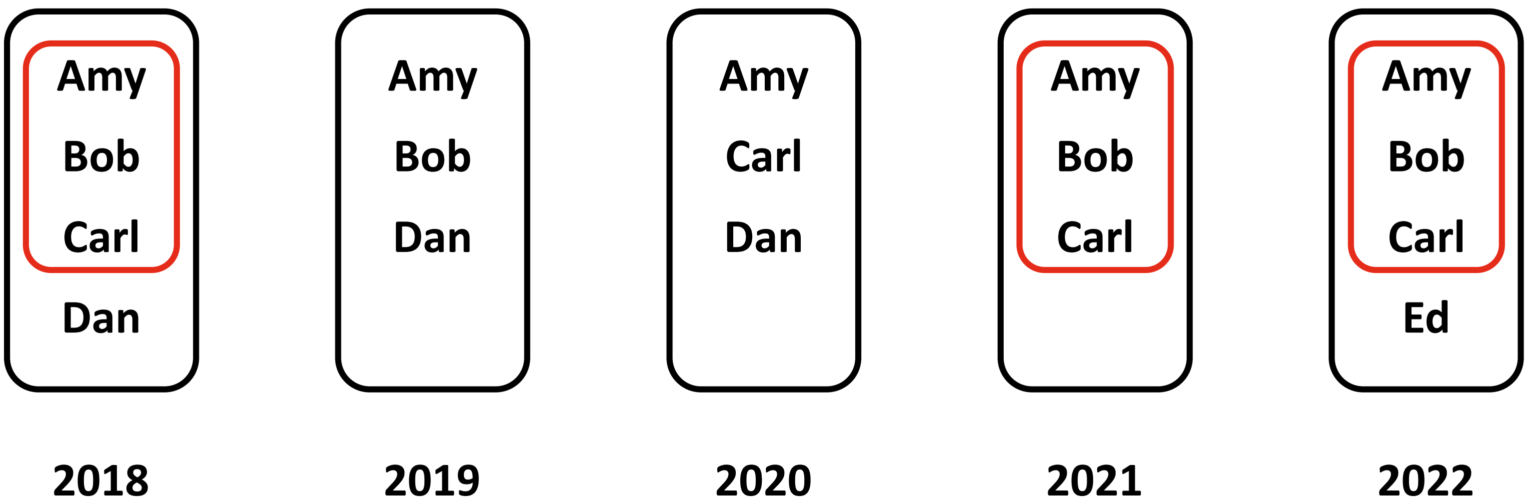



(a) Ordinary Graph (b) Hypergraph

- ➢ Sender and receivers of an email
- ➢ Co-authors of a publication
- ➢ Items co-purchased by a customer

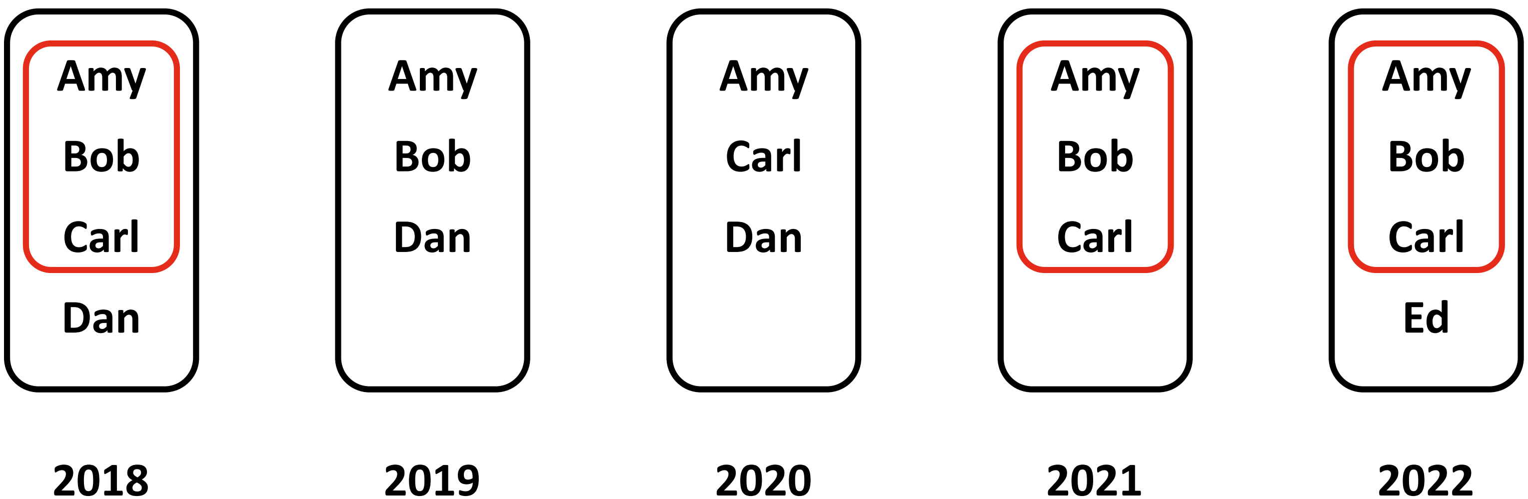
# Higher-Order Interaction (HOI)

- A **higher-order interaction (HOI)** is the **co-appearance** of a set of nodes in any hyperedge
  - ➢ E.g.) If $A$, $B$, and $C$ publish a paper together, any of $\{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}$ becomes a HOI



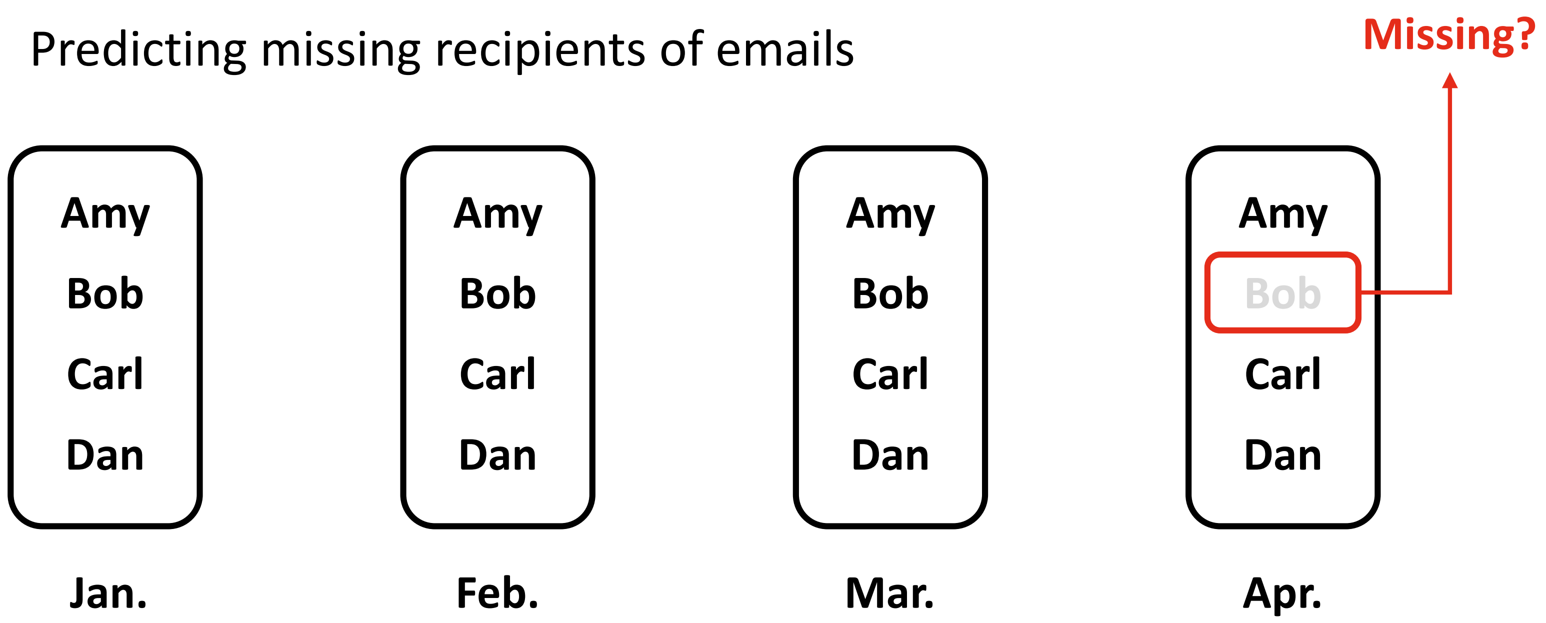
# Persistence of HOIs

- HOIs can appear **repeatedly** over time
- **Persistence** of repeated HOIs can be used to measure the strength or robustness of group relations



# Applications

- Predicting the persistence of HOIs has many **potential applications**
  - Recommending groups (e.g., Facebook groups) in social networks
  - Recommending multiple items together
  - Predicting missing recipients of emails



# Our Questions

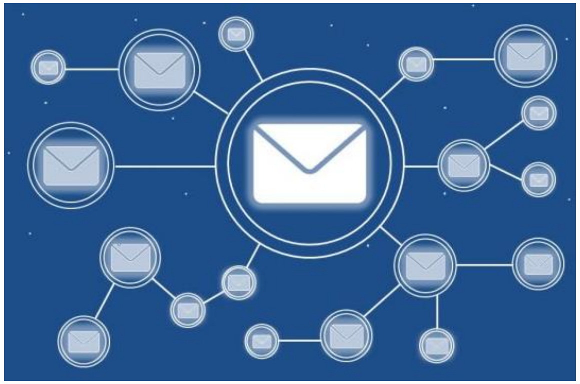
1. How do HOIs in real-world hypergraphs **persist over time**?
2. What are the **key factors** governing the persistence?
3. How accurately can we **predict** the persistence?

# Roadmap

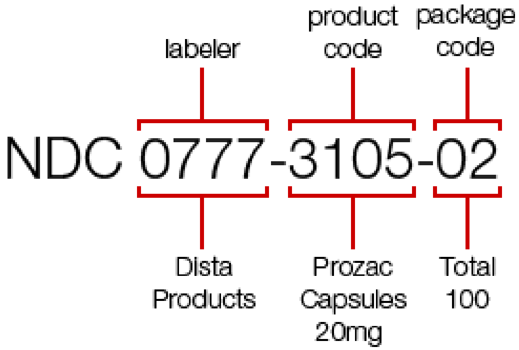
- Introduction
- **Observations <<**
  - Hypergraph-Level Analysis
  - Group-Level Analysis
  - Node-Level Analysis
- Predictions
- Conclusions

# Datasets

**Coauthorship**



**Email**




**NDC**

#boot
#networking
#drivers
#server
#wireless

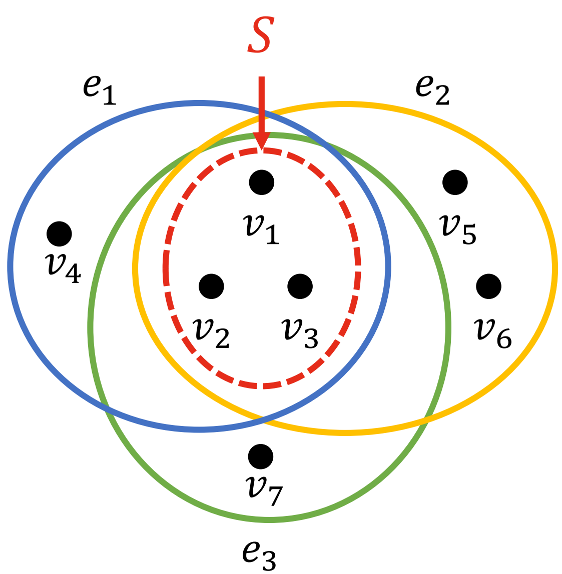
**Tags**

# Datasets

| Domain | Dataset | Node | Hyperedge | Time Unit |
|---|---|---|---|---|
| Coauthorship | DBLP<br>Geology<br>History | an author | authors | 1 Year |
| Contact | High<br>Primary | a person | a group interaction | 1 Day<br>6 Hours |
| Email | Enron<br>Eu | an email address | sender and all receivers | 1 Month<br>2 Weeks |
| NDC | Classes | a class label | class labels applied to a drug | 2 Years |
| | Substances | a substance | substances in a drug | |
| Tags | Math.sx<br>Ubuntu | a tag | tags added to a question | 1 Month |
| Threads | Math.sx<br>Ubuntu | a user | users who participate in a thread | 1 Month |

# Timestamped Hyperedges

- For each HOI $S$,
  - $E(S)$: Set of hyperedges containing $S$
  - $E(S,t)$: Set of hyperedges at time $t$ containing $S$
    - ➢ Hyperedge $e_i$ is associated with the timestamp $t_i$




**Timestamped Hyperedges:**

$e_1 = \{v_1, v_2, v_3, v_4\}$, $t_1 = 1$
$e_2 = \{v_1, v_2, v_3, v_5, v_6\}$, $t_2 = 1$
$e_3 = \{v_1, v_2, v_3, v_7\}$, $t_3 = 3$

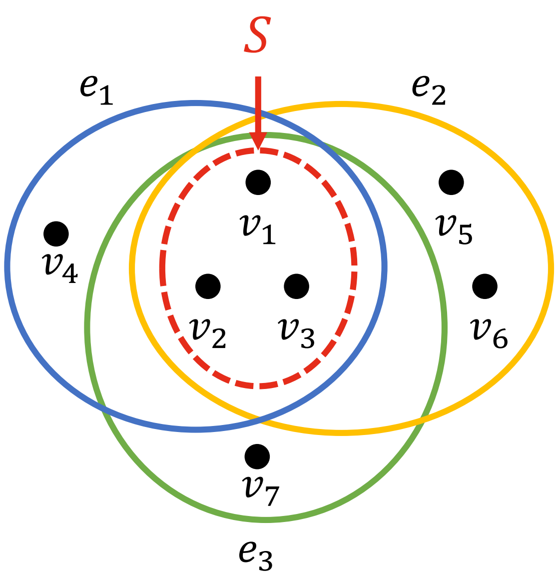
**Examples:**

$S = \{v_1, v_2, v_3\}$
$E(S) = \{e_1, e_2, e_3\}$
$E(S,1) = \{e_1, e_2\}$
$E(S,2) = \emptyset$
$E(S,3) = \{e_3\}$

# Measure: Persistence of a HOI

- **Persistence** of a HOI $S$ over a time range $T$ is the number of time units in $T$ when $S$ co-appear in any hyperedge, i.e.,

$$P(S,T) \coloneqq \sum_{t\in T} I(S,t) \qquad \text{where } I(S,t) = \begin{cases} 1, & if\ |E(S,t)| \geq 1 \\ 0, & otherwise \end{cases}$$




$E(S,1) = \{e_1, e_2\}$
$E(S,2) = \emptyset$
$E(S,3) = \{e_3\}$

$$\boldsymbol{P(S,[1,3]) = \sum_{t=1}^{3} I(S,t) = 1+0+1=2}$$

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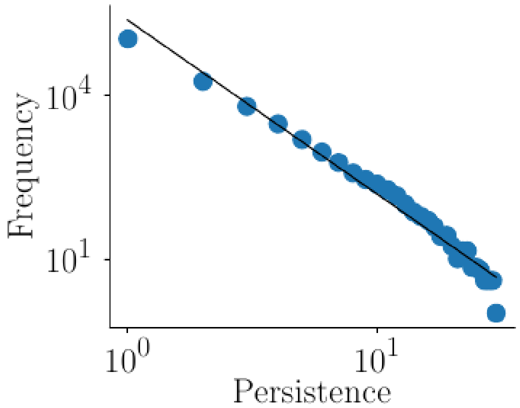
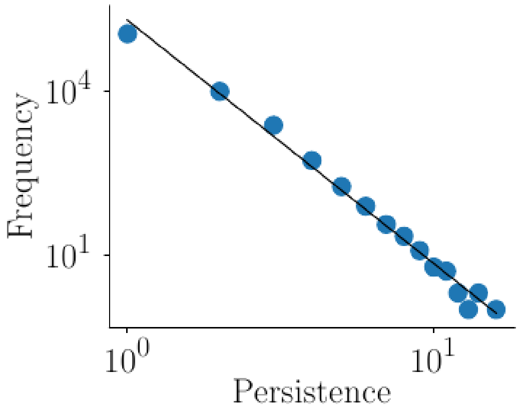
# Persistence vs. Frequency

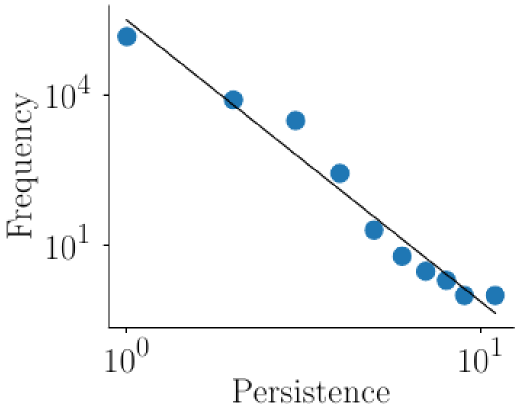
**Obs. 1:** Persistence of HOIs tends to follow a **power-law**.




DBLP ($|S| = 2$)




DBLP ($|S| = 3$)




DBLP ($|S| = 4$)

| | $R^2$ **of Fitted Line** | | |
|---|---|---|---|
| Size of HOIs | 2 | 3 | 4 |
| **Average over all 13 datasets** | **0.90** | **0.90** | **0.90** |

# Persistence vs. Size of HOIs

**Obs. 2:** As HOIs grow in size, their **average persistence** and the **power-law exponents** of fitted power-law distributions tend to decrease.

| Dataset | Average Persistence (Relative) | | | Power-Law Exponent (Relative) | | |
|---|---|---|---|---|---|---|
| Size of HOIs | 2 | 3 | 4 | 2 | 3 | 4 |
| **Average over all 13 datasets** | **1.00** | **0.72** | **0.63** | **1.00** | **0.71** | **0.59** |

# Roadmap

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# Group Features vs. Group Persistence

- We examined the relations between the structural group features and the persistence of HOIs (i.e., group persistence)
- We measured the **Pearson correlation coefficient (CC)** and **normalized mutual information (MI)** between the persistence and each structural feature to examine the relation between them
  - Normalized mutual information scales from 0 (no mutual information) to 1 (perfect correlation)

# Group Features: Definition

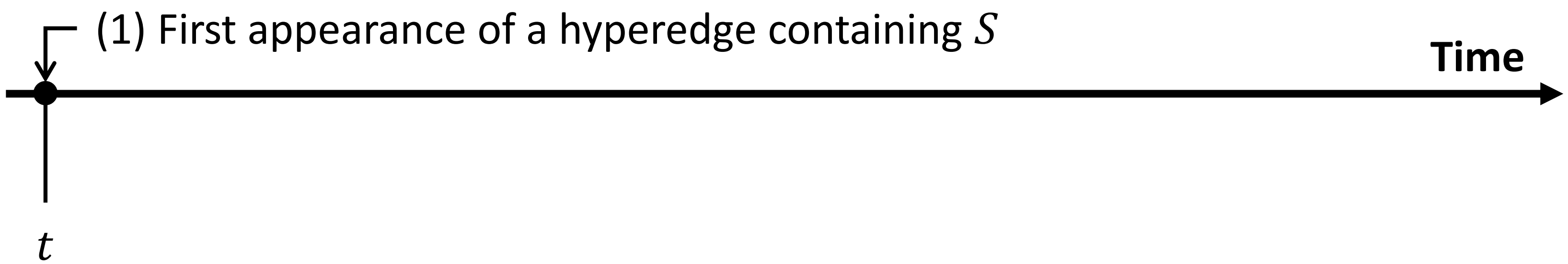
- Basic structural features of each HOI $S$:
  - $\#$: number of hyperedges including $S$
  - $\Sigma$: sum of sizes of hyperedges containing $S$
  - $\cup$: number of hyperedges overlapping $S$
  - $\Sigma \cup$: sum of sizes of hyperedges overlapping $S$
  - $\cap$: number of common neighbors of $S$
  - $\mathcal{H}$: entropy in sizes of hyperedges containing $S$
- Group structural features of each HOI $S$:
  - (1) $\#$, (2) $\#/\cup$, (3) $\Sigma/(\Sigma \cup)$, (4) $\cap$, (5) $\#/\cap$, (6) $\Sigma/\cap$, (7) $\Sigma/\#$, (8) $\mathcal{H}$

density of hyperedges containing $S$

avg. sizes of hyperedges containing $S$

# Measure: Structural Features & Persistence

1) HOI $S$ appears in a hyperedge for the first time at time $t$

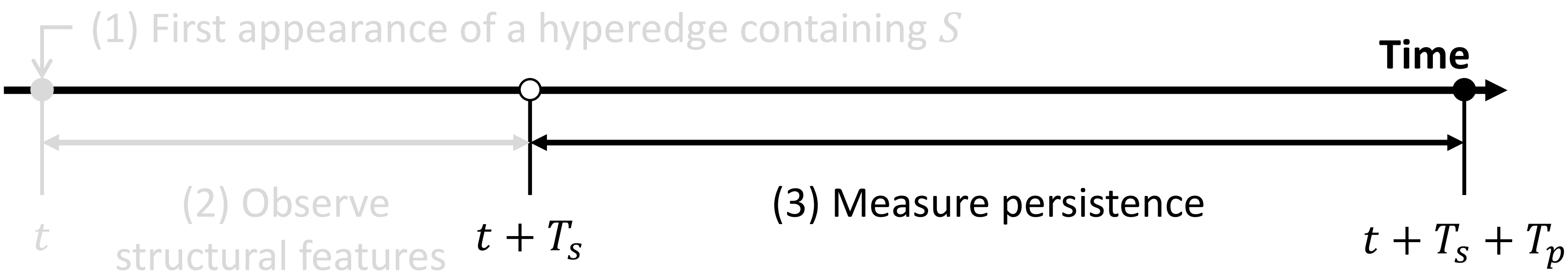


# Measure: Structural Features & Persistence

1) HOI $S$ appears in a hyperedge for the first time at time $t$
2) Compute its structural features using only the hyperedges appearing between time $t+1$ and $t+T_s$

# Measure: Structural Features & Persistence

1) HOI $S$ appears in a hyperedge for the first time at time $t$
2) Compute its structural features using only the hyperedges appearing between time $t+1$ and $t+T_s$
3) Measure its persistence between time $t+T_s+1$ and $t+T_s+T_p$




- We set $T_s=5$ and $T_p=10$

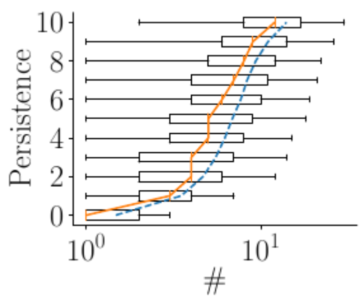
# Group Features vs. Group Persistence

**Obs. 3:** Persistence of each HOI S is positively correlated with (a) the **number of hyperedges containing $S$** and (b) the **entropy in the sizes of hyperedges containing $S$**.

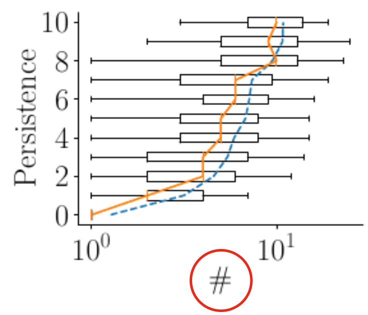
| | Size of HOIs | # | $\frac{\#}{\cup}$ | $\frac{\Sigma}{\Sigma \cup}$ | $\cap$ | $\frac{\#}{\cap}$ | $\frac{\Sigma}{\cap}$ | $\frac{\Sigma}{\#}$ | $\mathcal{H}$ |
|---|---|---|---|---|---|---|---|---|---|
| MI | 2 | 0.13 | 0.11 | 0.14 | 0.05 | 0.10 | 0.12 | 0.10 | **0.15** |
| | 3 | 0.11 | 0.06 | 0.08 | 0.05 | 0.08 | 0.09 | 0.08 | **0.12** |
| | 4 | 0.11 | 0.05 | 0.07 | 0.06 | 0.07 | 0.10 | 0.07 | **0.12** |
| | Avg. | 0.12 | 0.08 | 0.10 | 0.05 | 0.08 | 0.11 | 0.08 | **0.13** |
| CC | 2 | **0.36** | 0.09 | 0.09 | 0.17 | 0.19 | 0.26 | -0.08 | 0.32 |
| | 3 | **0.31** | 0.10 | 0.10 | 0.05 | 0.16 | 0.20 | -0.09 | 0.25 |
| | 4 | **0.30** | 0.13 | 0.13 | -0.01 | 0.17 | 0.20 | -0.10 | 0.24 |
| | Avg. | **0.32** | 0.10 | 0.11 | 0.07 | 0.17 | 0.22 | -0.09 | 0.27 |

# Group Features vs. Group Persistence

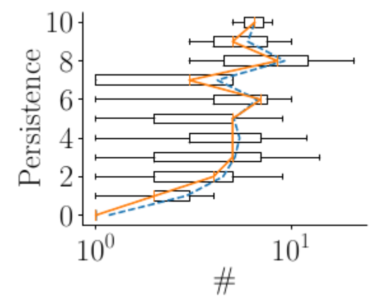
**Obs. 3:** Persistence of each HOI S is positively correlated with (a) the **number of hyperedges containing $S$** and (b) the **entropy in the sizes of hyperedges containing $S$**.



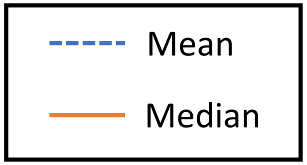
DBLP ($|S| = 2$)

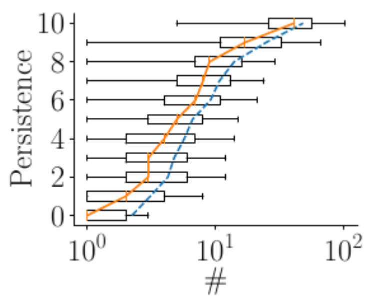


DBLP ($|S| = 3$)

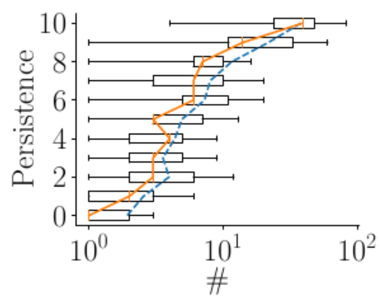


DBLP ($|S| = 4$)

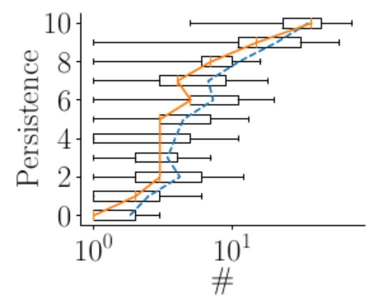


Eu ($|S| = 2$)



Eu ($|S| = 3$)



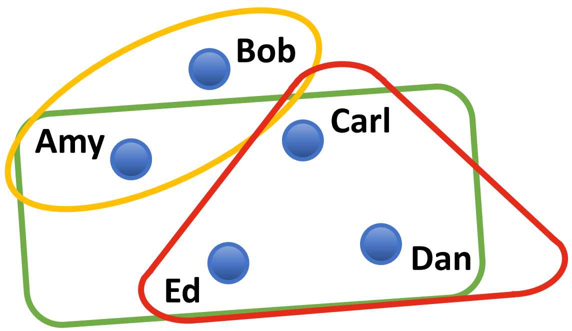
Eu ($|S| = 4$)

# Node Features: Definition

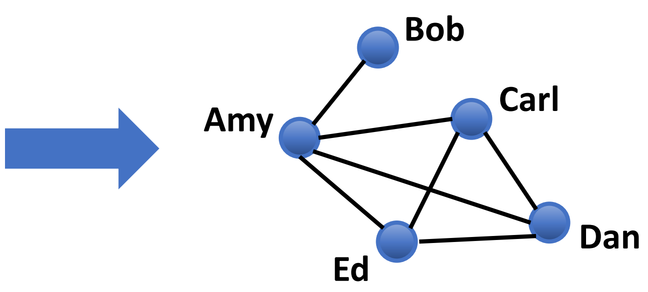
- We examine the relations between the persistence of each HOI (i.e., group persistence) and the structural features of individual nodes involved in the HOI
- Structural features of each node $v$ in the clique expansion:
  a. **degree** $d(v)$
  b. **weighted degree** $w(v)$
  c. **core number** $c(v)$
  d. **PageRank** $r(v)$
  e. **average degree of neighbors** $\bar{d}(v)$
  f. **average weighted degree of neighbors** $\overline{w}(v)$
  g. **local clustering coefficient** $l(v)$
  h. **number of occurrences of $v$** $o(v)$

# Clique Expansion: Definition

- The **clique expansion** of a hypergraph is a pairwise graph between nodes
- It is obtained by replacing each hyperedge with the clique with the nodes in the hyperedge




Hypergraph




Clique Expansion

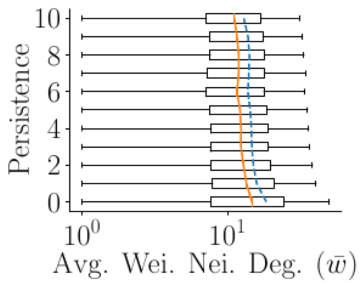
# Node Features vs. Group Persistence

**Obs. 4:** Persistence of each HOI S is negatively correlated with the **average (weighted) degree of neighbors** of each node involved in the HOI.

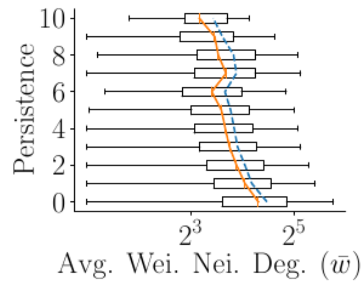
| | Size of HOIs | $d$ | $w$ | $c$ | $r$ | $\bar{d}$ | $\bar{w}$ | $l$ | $o$ |
|---|---|---|---|---|---|---|---|---|---|
| MI | 2 | 0.04 | 0.09 | 0.04 | **0.17** | 0.16 | 0.17 | 0.15 | 0.08 |
| | 3 | 0.03 | 0.06 | 0.04 | 0.09 | 0.09 | **0.10** | 0.09 | 0.05 |
| | 4 | 0.03 | 0.05 | 0.06 | 0.07 | 0.07 | **0.07** | 0.07 | 0.04 |
| | Avg. | 0.04 | 0.07 | 0.05 | 0.11 | 0.11 | **0.11** | 0.10 | 0.05 |
| CC | 2 | 0.05 | 0.09 | -0.01 | 0.07 | -0.12 | **-0.14** | -0.08 | 0.09 |
| | 3 | -0.02 | 0.06 | -0.05 | 0.03 | -0.11 | **-0.12** | -0.02 | 0.05 |
| | 4 | -0.07 | 0.03 | -0.09 | 0.03 | **-0.14** | -0.14 | 0.03 | 0.00 |
| | Avg. | -0.01 | 0.06 | -0.05 | 0.04 | -0.12 | **-0.13** | -0.02 | 0.05 |

# Node Features vs. Group Persistence

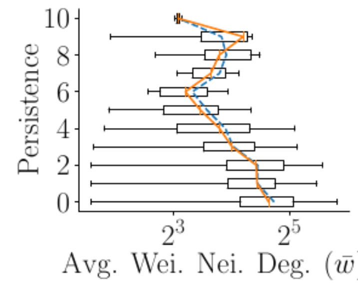
**Obs. 4:** Persistence of each HOI S is negatively correlated with the **average (weighted) degree of neighbors** of each node involved in the HOI.



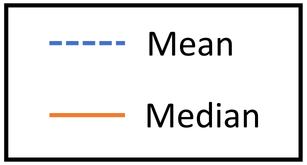

DBLP ($|S| = 2$)

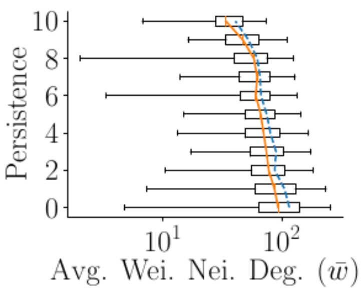



DBLP ($|S| = 3$)




DBLP ($|S| = 4$)

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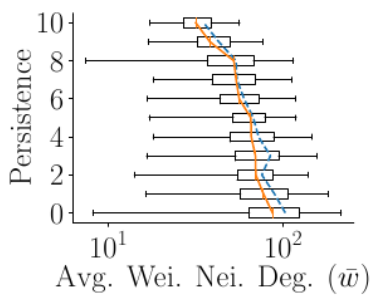


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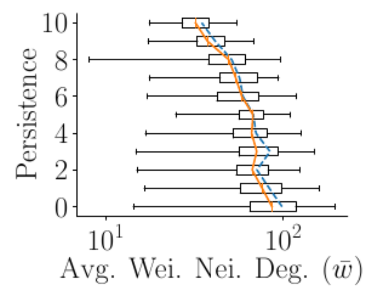


Eu ($|S| = 4$)

# Roadmap

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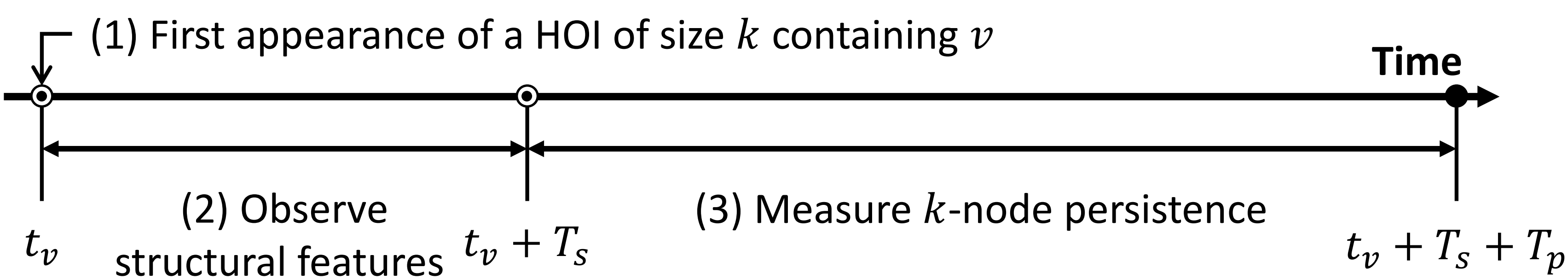
# Node Features vs. Node Persistence

- We explore the relations between the structural features of each node and its $k$-node persistence
- **$k$-node persistence** of a node $v$: average persistence of the HOIs of size $k \in \{2,3,4\}$ that the node $v$ is involved in
- For each node $v$, let $t_v$ be the time when $v$ is involved in any HOI of size $k$ for the first time
  - Measure the structural node features of $v$ using only the hyperedges appearing between time $t_v + 1$ and $t_v + T_S$



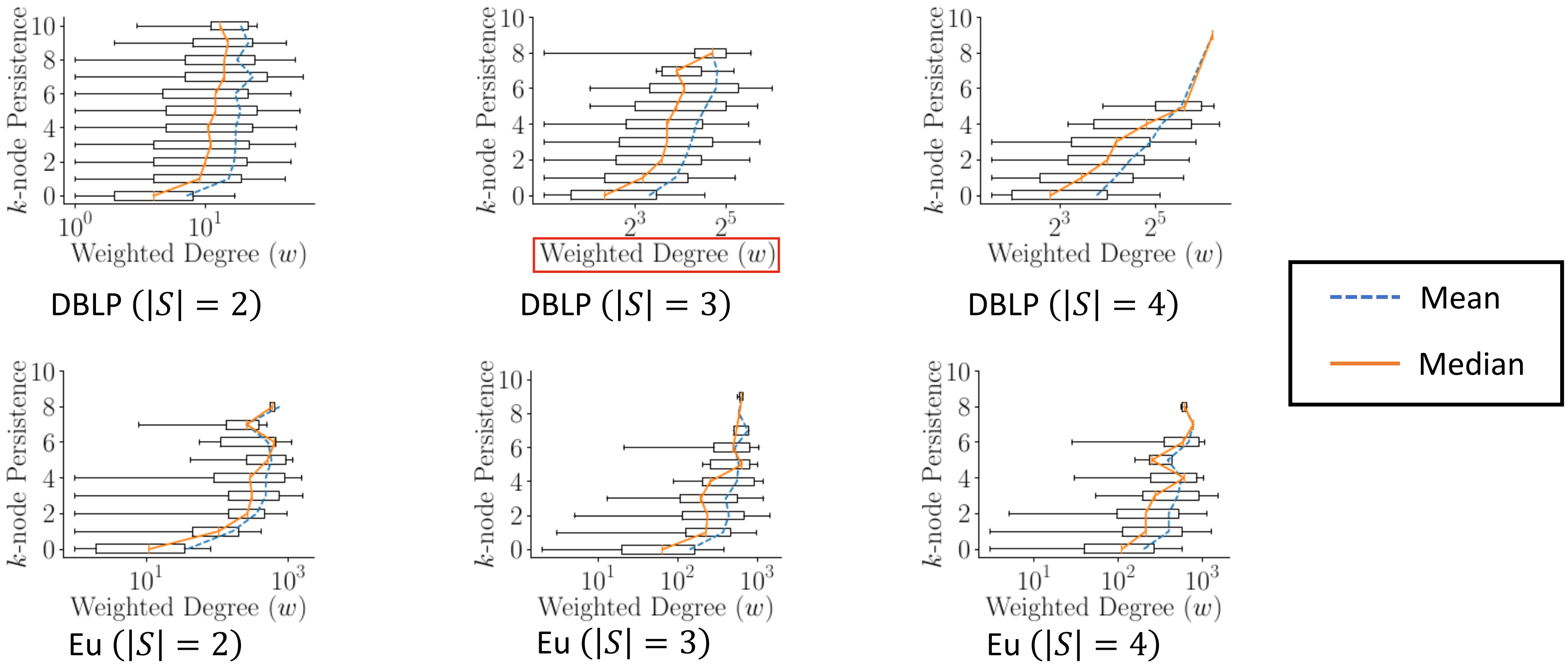
# Node Features vs. Node Persistence

**Obs. 5:** The **weighted degree** and **number of occurrences** of each node are positively correlated with the $k$-node persistence of HOIs that the node is involved in.

| | Size of HOIs | $d$ | $w$ | $c$ | $r$ | $\bar{d}$ | $\bar{w}$ | $l$ | $o$ |
|---|---|---|---|---|---|---|---|---|---|
| MI | 2 | 0.35 | 0.43 | 0.28 | **0.53** | 0.49 | 0.51 | 0.43 | 0.41 |
| | 3 | 0.30 | 0.37 | 0.24 | **0.44** | 0.42 | 0.44 | 0.37 | 0.34 |
| | 4 | 0.26 | 0.31 | 0.21 | **0.36** | 0.35 | 0.36 | 0.31 | 0.30 |
| | Avg. | 0.30 | 0.37 | 0.24 | **0.44** | 0.42 | 0.43 | 0.37 | 0.35 |
| CC | 2 | 0.15 | 0.22 | 0.14 | 0.08 | 0.00 | -0.07 | -0.02 | **0.26** |
| | 3 | 0.04 | 0.16 | 0.04 | 0.03 | -0.04 | -0.08 | -0.04 | **0.17** |
| | 4 | 0.03 | 0.12 | 0.01 | 0.02 | -0.05 | -0.07 | -0.04 | **0.13** |
| | Avg. | 0.07 | 0.17 | 0.06 | 0.04 | -0.03 | -0.07 | -0.03 | **0.19** |

# Node Features vs. Node Persistence

**Obs. 5:** The **weighted degree** and **number of occurrences** of each node are positively correlated with the $k$-node persistence of HOIs that the node is involved in.



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# Prediction Experiments

- **Exp. 1: Predictability.** How accurately can we predict the persistence of HOIs using the structural features?
- **Exp. 2: Feature Importance.** Which structural features are important in predicting the persistence?
- **Exp. 3: Effect of Observation Periods.** How does the period of observation for measuring the structural features affect the prediction accuracy?

# Problem 1: Persistence Prediction

- **Given**:
  - a **HOI $S$** that appears for the first time at time $t$,
  - all **hyperedges appearing in the past**
    - between time $t + 1$ and $t + T_s$
- **Predict**:
  - **persistence of $S$ in the near future**
    - between $t + T_s + 1$ and $t + T_s + T_p$

# Problem 2: $k$-Node Persistence Prediction

- **Given**:
  - a **node $v$** involved in a HOI of size $k$ for the first time at time $t$,
  - all **hyperedges appearing in the past**
    - between time $t+1$ and $t+T_s$
- **Predict**:
  - **$k$-node persistence of $v$ in the near future**
    - between $t+T_s+1$ and $t+T_s+T_p$

# Prediction Methods

- We use all 16 structural features (8 group and 8 node features) as input features into **four regression models**:
  1) multiple linear regression (**LR**)
  2) random forest regression (**RF**)
  3) linear support vector regression (**SVR**)
  4) multi-layer perceptron regressor (**MLP**)

  ✓ **Baseline:** mean ($k$-node) persistence in the training set
- Training set: **2/3** of the HOIs and their persistence and **4/5** of the nodes and their $k$-node persistence
- Test set: the remaining ones

# Evaluation Methods

- We evaluate the predictive performance of the models using two metrics:
  - **Coefficients of determination ($R^2$)**: measures how well the predictions approximate the real data
  - **Root mean squared error ($RMSE$)**: between predicted and real ($k$-node) persistence
- A **higher $R^2$** and **lower $RMSE$** indicate better performance

# Exp. 1: Predictability

**Obs. 6:** The **structural features are useful** for predicting the persistence, especially when the size of the HOI is large.

| Target | Prediction of Persistence of HOIs | | | | | | Prediction of $k$-Node Persistence of Nodes | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Measure | $R^{2*}$ | | | RMSE** | | | $R^{2*}$ | | | RMSE** | | |
| Size of HOIs | 2 | 3 | 4 | 2 | 3 | 4 | 2 | 3 | 4 | 2 | 3 | 4 |
| Mean | 0.00 | 0.00 | 0.00 | 1.29 | 0.73 | 0.60 | -0.01 | -0.01 | -0.03 | 0.75 | 0.56 | 0.54 |
| SVR | 0.17 | 0.13 | 0.10 | 1.12 | 0.63 | 0.48 | 0.03 | 0.01 | 0.00 | 0.73 | 0.56 | 0.54 |
| LR | 0.28 | 0.22 | 0.23 | 1.05 | 0.58 | 0.45 | <u>0.17</u> | <u>0.15</u> | <u>0.09</u> | <u>0.75</u> | <u>0.71</u> | <u>0.67</u> |
| MLP | <u>0.34</u> | <u>0.31</u> | <u>0.37</u> | <u>0.95</u> | <u>0.53</u> | <u>0.42</u> | 0.14 | 0.06 | 0.02 | 0.77 | 0.75 | 0.72 |
| RF | **0.61** | **0.62** | **0.68** | **0.83** | **0.38** | **0.24** | **0.61** | **0.66** | **0.71** | **0.54** | **0.41** | **0.39** |

*The higher, the better. **The lower, the better.

# Measure: Feature Importance

- We use the **Gini importance** to measure the importance of each structural feature for random forest
- We compute the **rankings** of the features based on the importance

# Exp. 2: Feature Importance

**Obs. 7:** In predicting the persistence, the **number of hyperedges containing $S$ (i.e., $\#$)**, and the **average (weighted) degree of the neighbors of each node in $S$ (i.e., $\bar{w}$ and $\bar{d}$)** are most useful.

| Size of HOIs | $\#$ | $\frac{\#}{\cup}$ | $\frac{\Sigma}{\Sigma\cup}$ | $\cap$ | $\frac{\#}{\cap}$ | $\frac{\Sigma}{\cap}$ | $\frac{\Sigma}{\#}$ | $\mathcal{H}$ | $d$ | $w$ | $c$ | $r$ | $\bar{d}$ | $\bar{w}$ | $l$ | $o$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | **2.8** | 10.7 | 8.6 | 13.1 | 13.3 | 9.0 | 9.2 | 8.7 | 9.9 | 8.6 | 8.8 | 5.9 | *4.9* | $\underline{4.3}$ | 6.4 | 11.9 |
| 3 | *5.4* | 9.2 | 9.2 | 11.8 | 11.2 | 9.6 | 9.8 | 7.9 | 11.2 | 9.1 | 8.4 | 5.7 | $\underline{5.1}$ | **4.3** | 6.4 | 12.0 |
| 4 | **5.3** | 9.3 | 9.9 | 10.3 | 10.6 | 8.3 | 8.7 | *7.0* | 9.5 | 7.3 | 9.2 | 7.7 | 7.7 | $\underline{6.3}$ | 8.0 | 11.0 |
| Avg. | **4.5** | 9.7 | 9.2 | 11.7 | 11.7 | 9.0 | 9.2 | 7.9 | 10.2 | 8.3 | 8.8 | 6.4 | *5.9* | $\underline{5.0}$ | 6.9 | 11.6 |

Feature Importance Ranking

# Exp. 2: Feature Importance

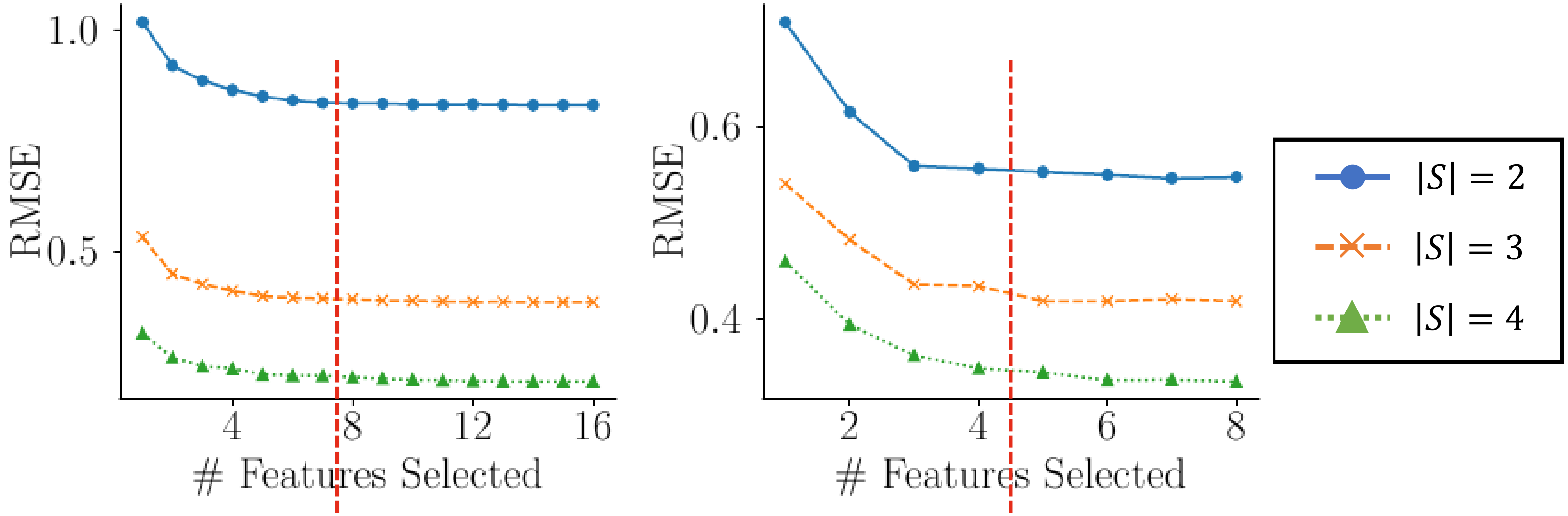
**Obs. 8:** In predicting the $k$-node persistence, its **PageRank (i.e., $r$)** and the **average (weighted) degree of its neighbors (i.e., $\bar{w}$ and $\bar{d}$)** are most useful.

| **Size of HOIs** | $d$ | $w$ | $c$ | $r$ | $\bar{d}$ | $\bar{w}$ | $l$ | $o$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 6.7 | 4.3 | 7.2 | <u>3.2</u> | 3.4 | **2.9** | 5.3 | <u>3.2</u> |
| 3 | 6.6 | 4.1 | 7.3 | **2.7** | *3.5* | **2.7** | 5.0 | 4.3 |
| 4 | 6.1 | 4.0 | 6.6 | **2.6** | *3.5* | <u>3.1</u> | 5.3 | 4.9 |
| Avg. | 6.4 | 4.1 | 7.0 | **2.8** | *3.5* | <u>2.9</u> | 5.2 | 4.1 |

Feature Importance Ranking

# Exp. 2: Effect of Number of Features

**Obs. 9:** About a **half of the considered structural features** based on their importance yields similar performance.




(1) Persistence (2) $k$-Node Persistence

# Exp. 3: Effect of Observation Periods

**Obs. 10:** Observing HOIs for **longer periods of time** enables us to better predict their persistence.

| Target | Persistence of HOIs | | | | | | $k$-Node Persistence of Nodes | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Measure | RMSE* of RF | | | Improvement (in %) | | | RMSE* of RF | | | Improvement (in %) | | |
| $T_s$ | 2** | 3 | 4 | 2 | 3 | 4 | 2 | 3 | 4 | 2 | 3 | 4 |
| 1 | 0.96 | 0.48 | 0.32 | 31.6 | 42.3 | 50.7 | 0.62 | 0.46 | 0.43 | 18.5 | 25.5 | 31.8 |
| 3 | 0.88 | 0.42 | 0.28 | 34.1 | 45.4 | 55.0 | 0.55 | **0.41** | **0.38** | 24.8 | **29.1** | **34.4** |
| 5 | **0.83** | **0.38** | **0.24** | **36.0** | **47.7** | **59.4** | **0.54** | 0.41 | 0.39 | **27.4** | 26.4 | 27.5 |

*The lower, the better. **The size of HOIs (i.e., $|S|$).

# Roadmap

- Introduction
- Observations
  - Hypergraph-Level Analysis
  - Group-Level Analysis
  - Node-Level Analysis
- Predictions
- **Conclusions <<**

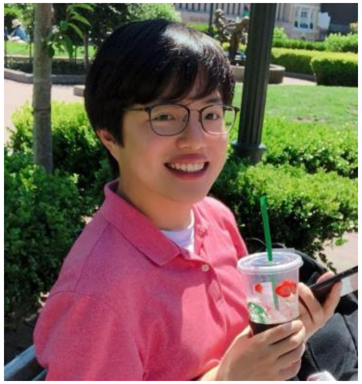
# Conclusions

- We empirically examined the **persistence of HOIs** at hypergraph-, group-, and node- levels in 13 real-world hypergraphs to answer the following questions:

✓ How is the persistence of HOIs **distributed**?
✓ Which **structural features** govern the persistence of HOIs?
✓ How accurately can we **forecast the persistence of HOIs?**

- Github link: https://github.com/jin-choo/persistence

# On the Persistence of Higher-Order Interactions in Real-World Hypergraphs



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