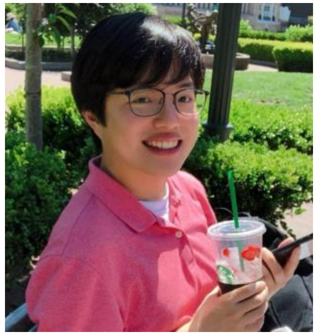
KAIST AI
Kim Jaechul Graduate School

ICDM 2022

Reciprocity in Directed Hypergraphs: Measures, Findings, and Generators



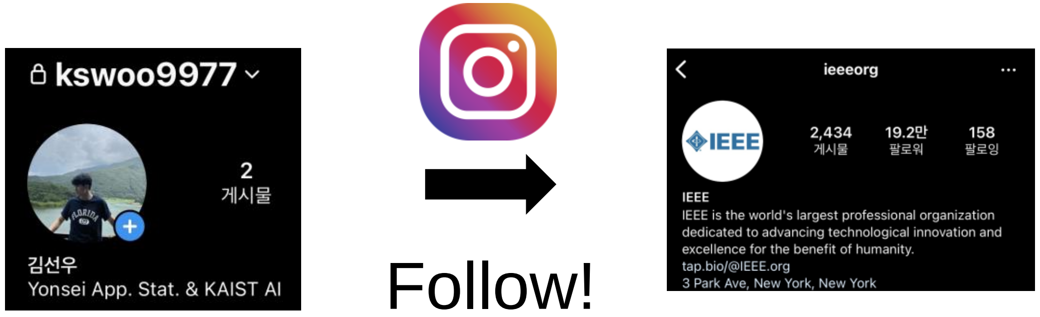
Sunwoo Kim | Minyoung Choe | Jaemin Yoo | Kijung Shin

Roadmap

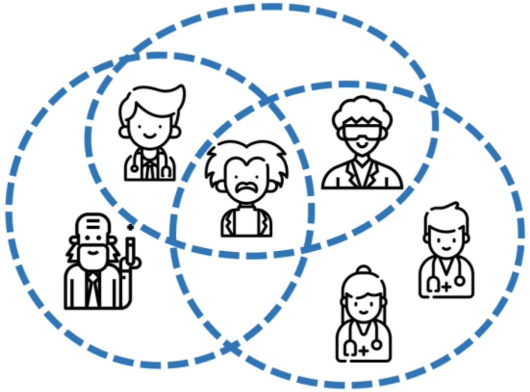
- Overview
- Reciprocity Measure
- Observations
- Generators
- Conclusions

What is Directed Hypergraph?

- In the real world, there are…
  - Directional relations: “Follow” in a social media
  - Group relations: Academic collaborations



Social Media

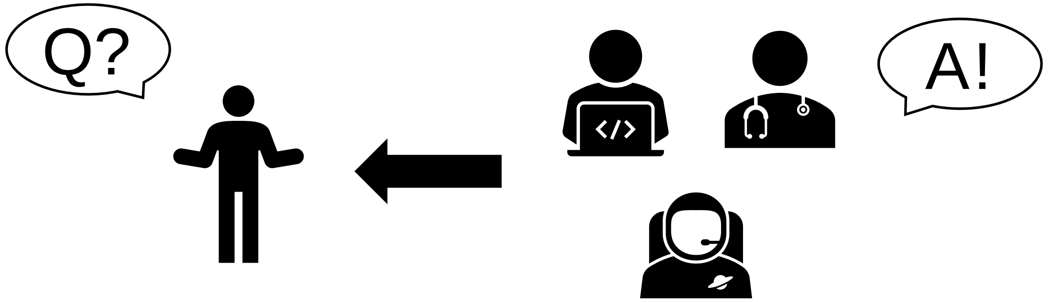


Collaborations of Researchers

What is Directed Hypergraph?

- Directional relations + Group relations = Directional group relations
  - Chemical reactions
  - Question & Answering

$$\begin{array}{ccc} CH_4 & & CO_2 \\ + & \longrightarrow & + \\ 2O_2 & & 2H_2O \end{array}$$



stackoverflow askubuntu

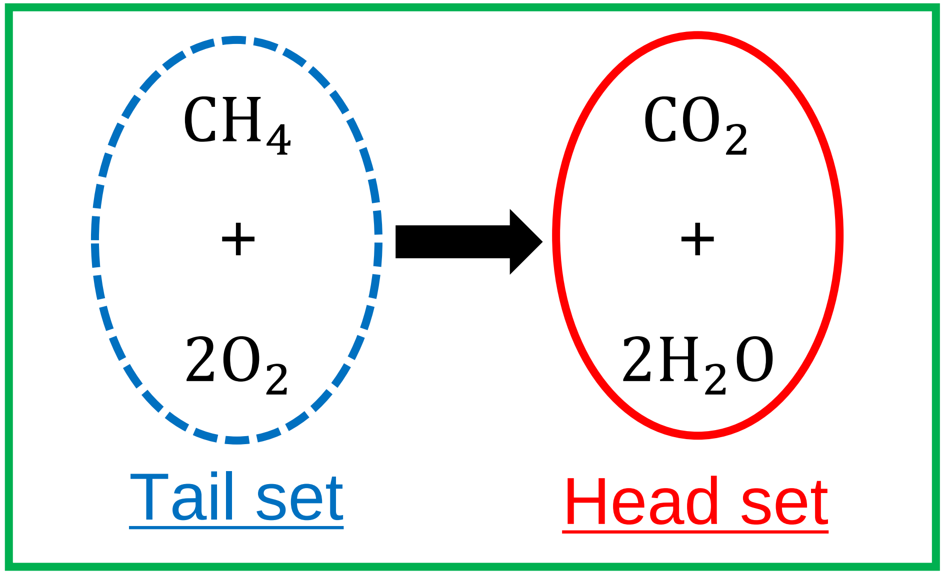
Chemical Reaction

Question & Answering

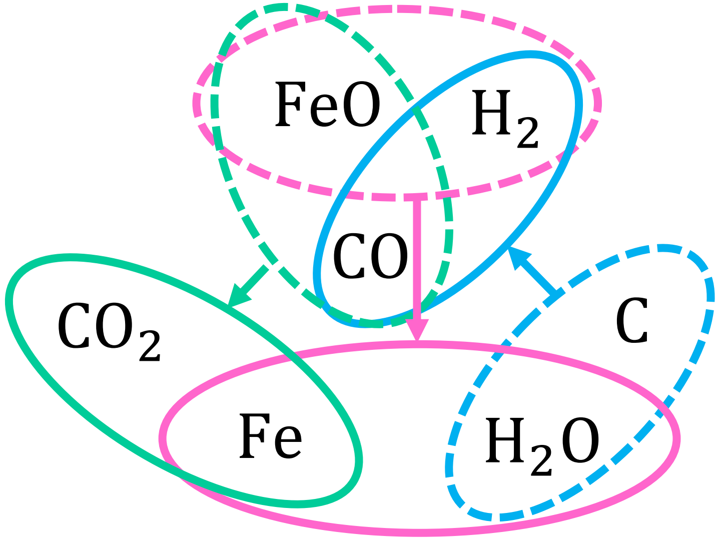
Modeling with Directed Hypergraphs!

What is Directed Hypergraph?

- Directed hypergraph consists of nodes and hyperarcs.
- Each hyperarc consists of two node sets: Head set and Tail set.
- Head set: Destination & Tail set: Source



Hyperarc

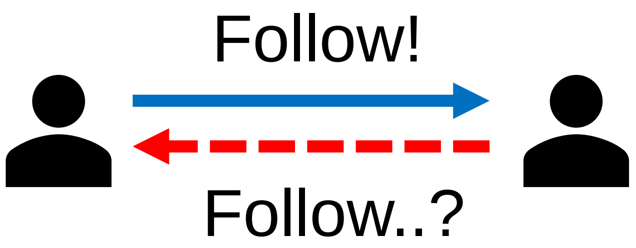


Directed Hypergraph

What is Reciprocity?

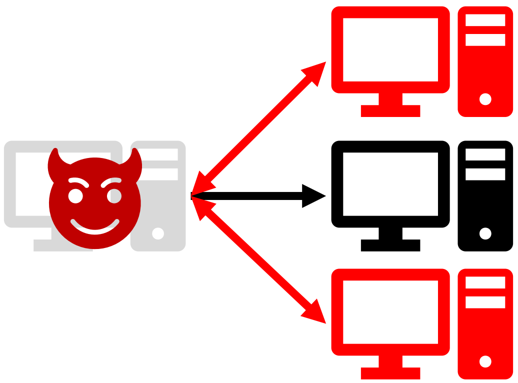
- How mutually are nodes linked?

In a graph: $\frac{\text{\# of Mutually Connected Edges}}{\text{\# of Edges}}$

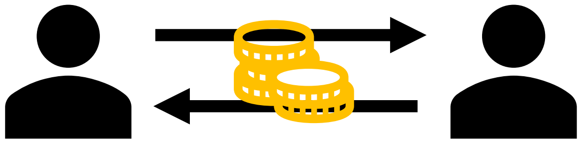


- Useful in understanding human interaction, computer virus spread model, and behavior (trust) prediction.

Social network



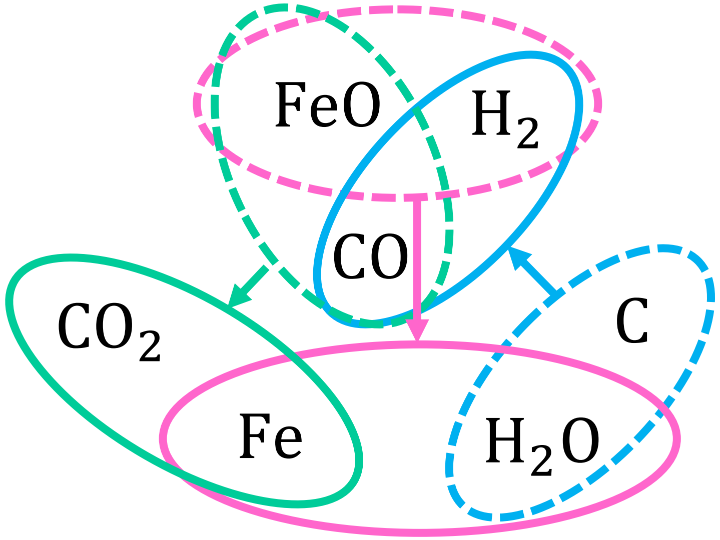
Virus spread

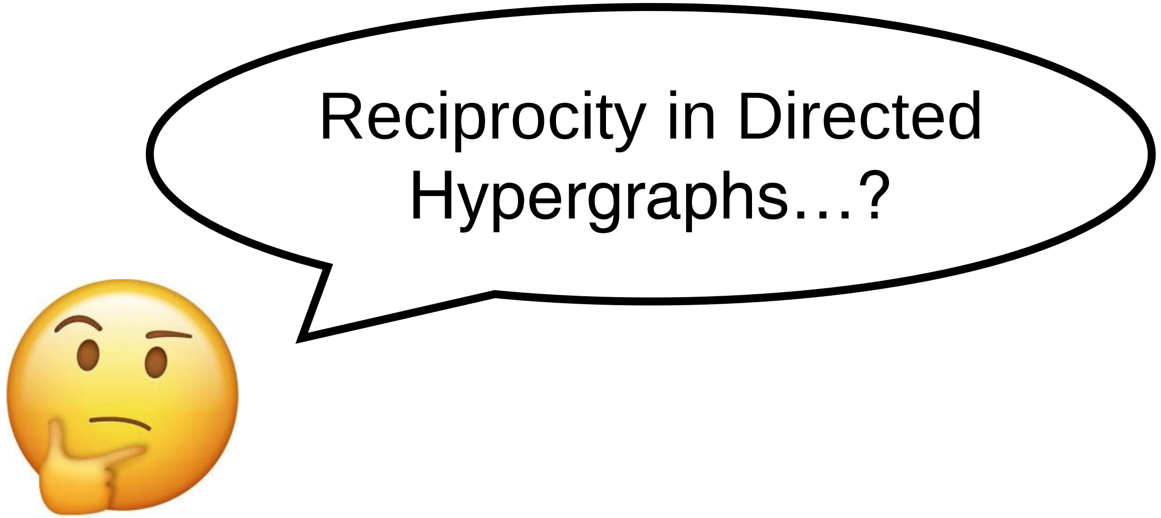


Behavior prediction

Our Research Question

- RQ1) How can we measure reciprocity in a directed hypergraph?
- RQ2) What are the real-world hypergraphs' reciprocal patterns?
- RQ3) Can we generate real-world hypergraphs' reciprocal patterns?





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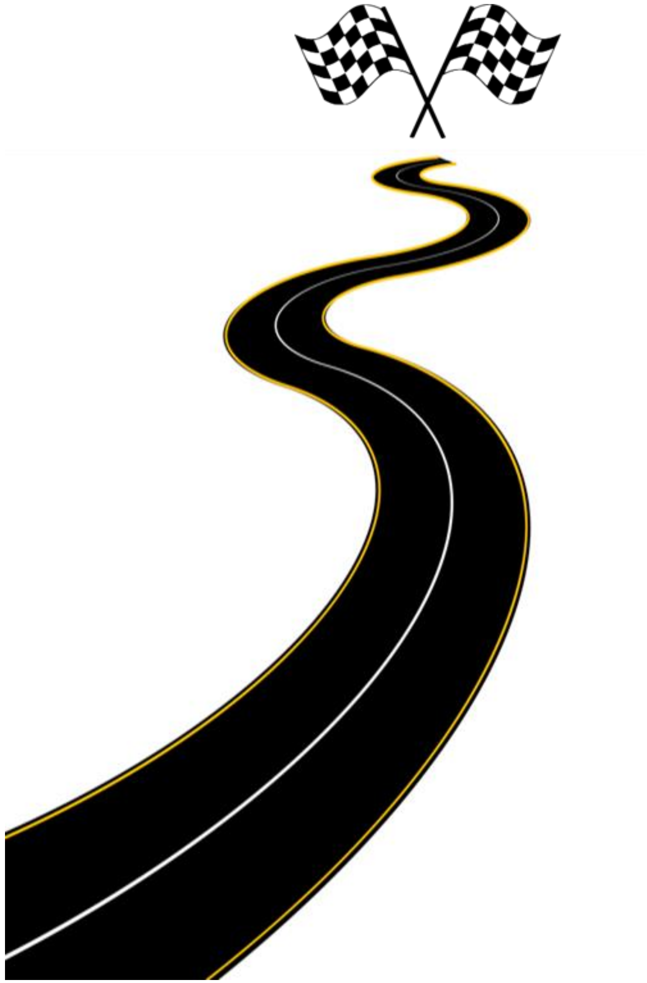
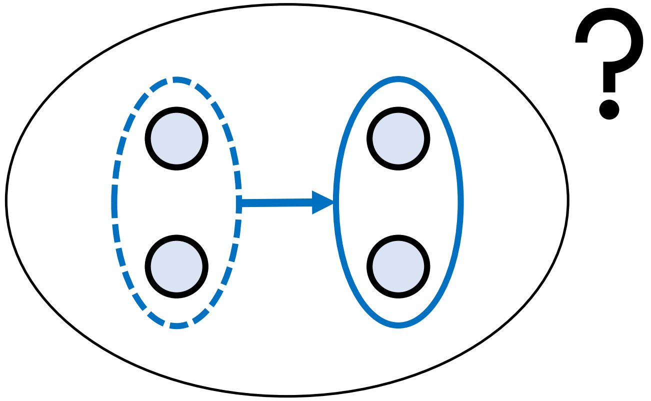


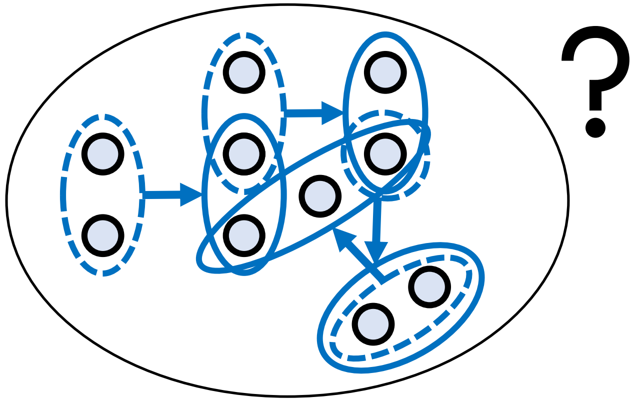
Overall Framework

AXIOM (8 axioms):
8 properties that a proper reciprocity measure should have.

HyperRec:
A hypergraph reciprocity measure that satisfies all the proposed axioms.

- We provide reciprocity measures for a hyperarc and a hypergraph.

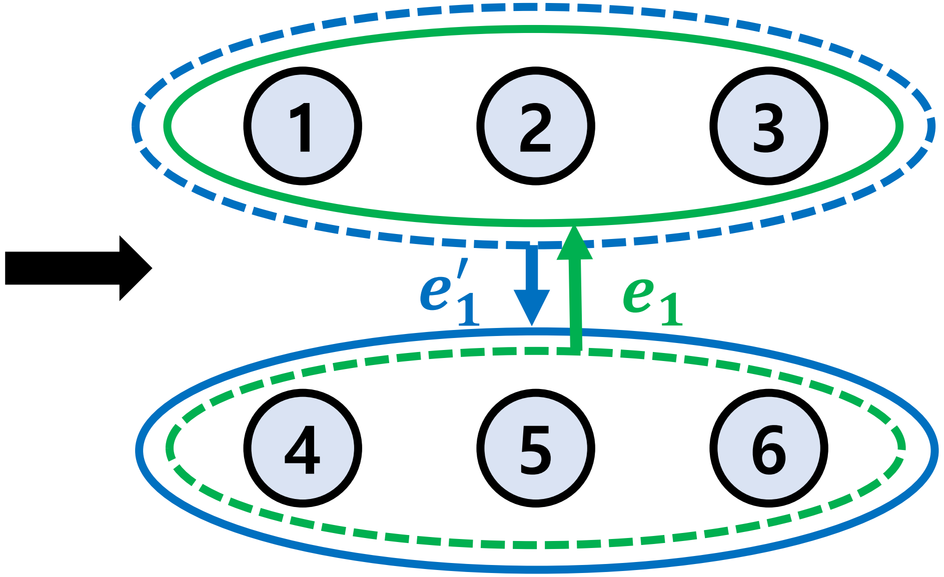


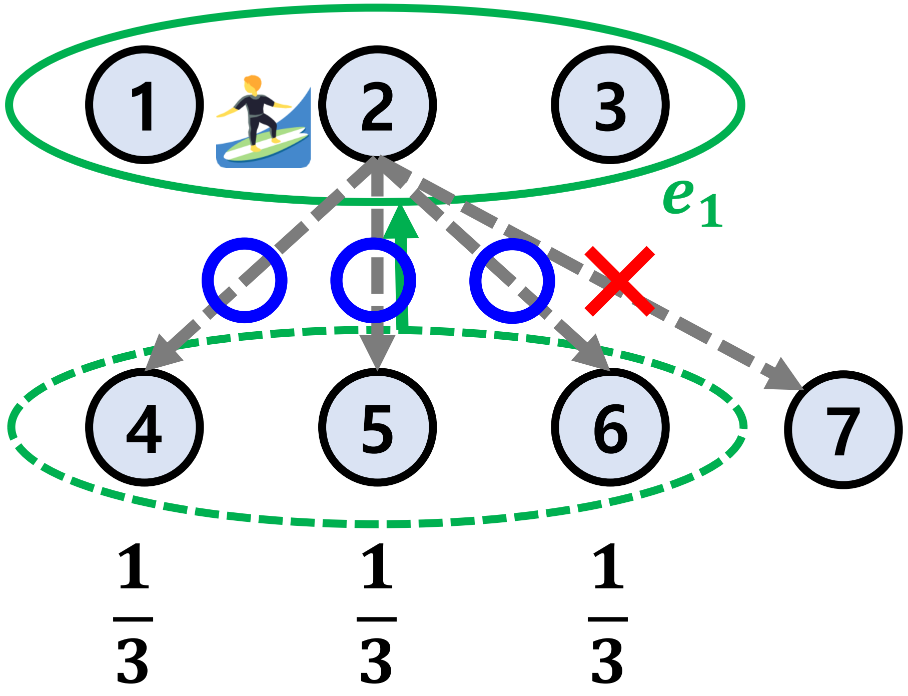


HyperRec: Proposed Reciprocity Measure

- HyperRec is based on the **random surfer's** return probability.
- Consider measuring the reciprocity of a specific hyperarc $\boldsymbol{e_1}$.
- Ideally, it should return to each of its sources with prob. 1/(tail set size).

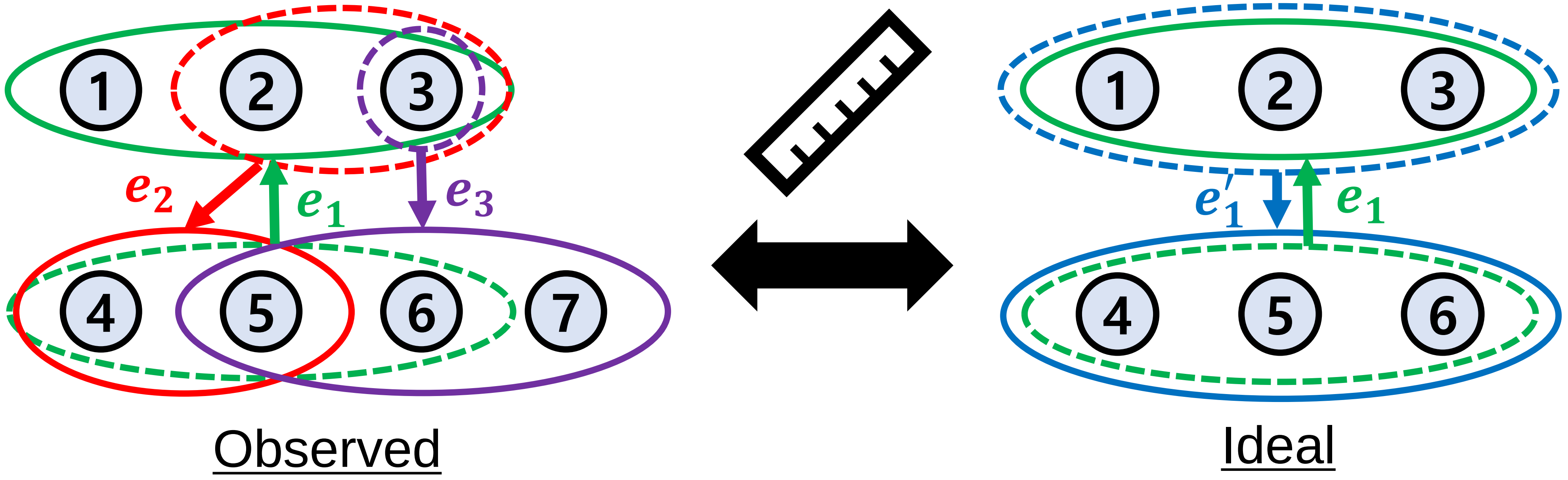
Optimally reciprocal hyperarc $\boldsymbol{e_1'}$





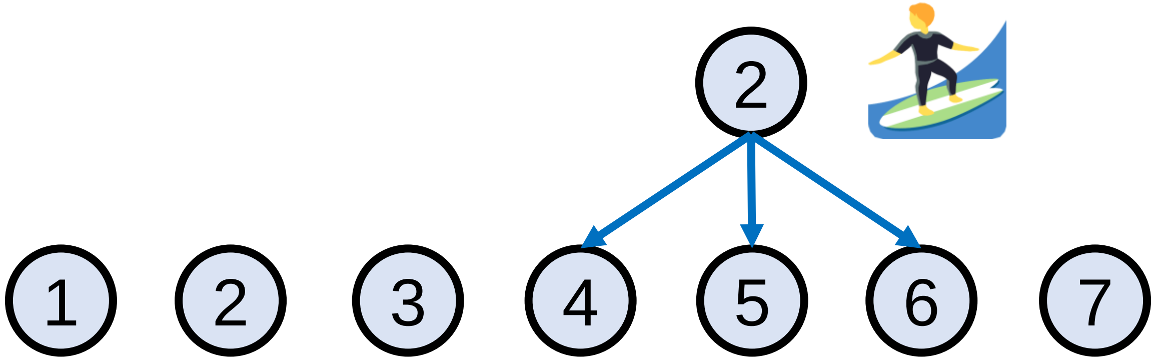
HyperRec: Proposed Reciprocity Measure

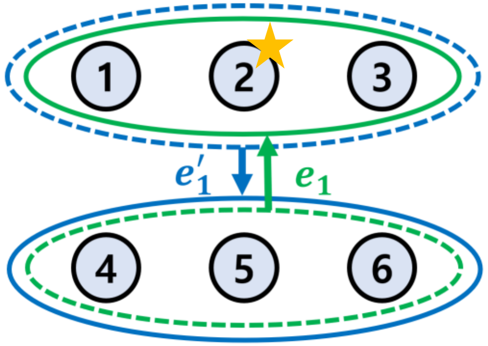
- There are observed hyperarcs (e_2, e_3).
- We use these hyperarcs (Reciprocal set) to measure the reciprocity of e_1.
- HyperRec measures how far observed hyperarcs are from the ideal case.
- Distance: Transition (return) probability distance between them.



HyperRec: Proposed Reciprocity Measure

- Transition (return) probability from node 2 to other nodes.





Ideal

Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0

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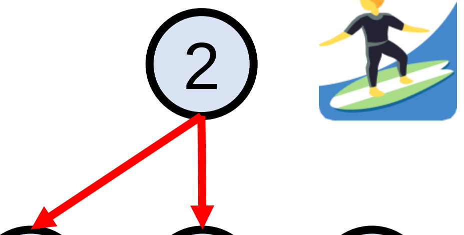
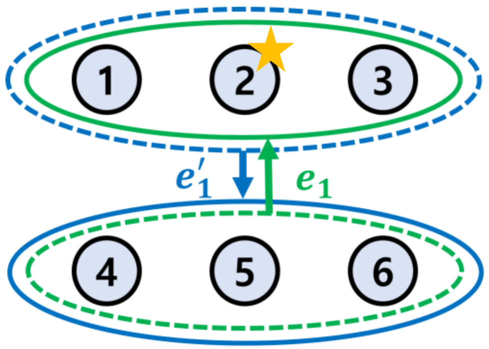
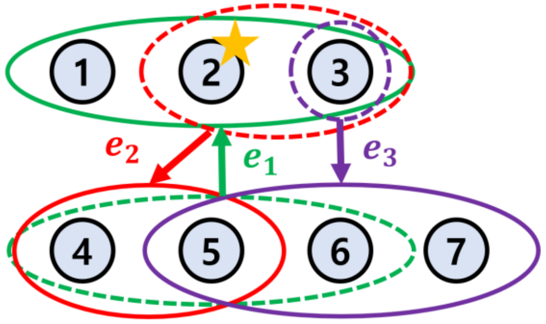


	1	2	3	4	5	6	7
Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
e_2	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0



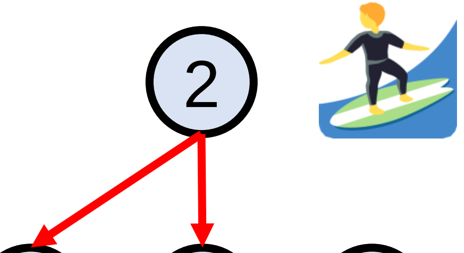
Ideal



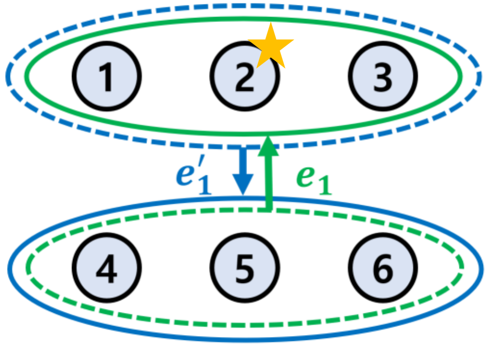
Observed

HyperRec: Proposed Reciprocity Measure

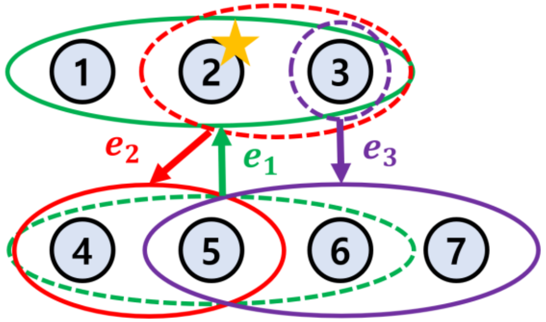
- Transition (return) probability from node 2 to other nodes.



Probability Distance		1	2	3	4	5	6	7
	Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
	e_2	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0



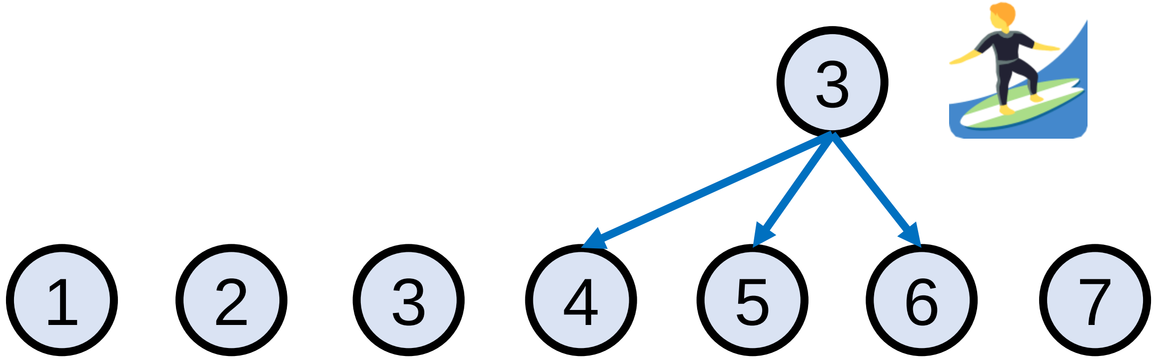
Ideal



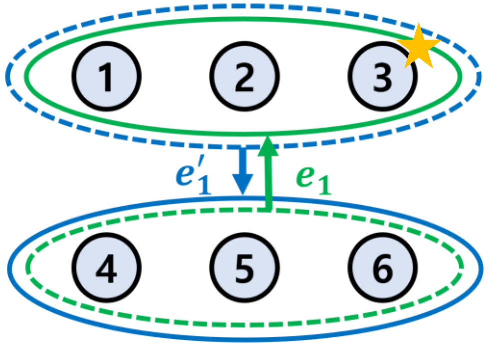
Observed

HyperRec: Proposed Reciprocity Measure

- Transition (return) probability from node 3 to other nodes.



Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0



Ideal

HyperRec: Proposed Reciprocity Measure

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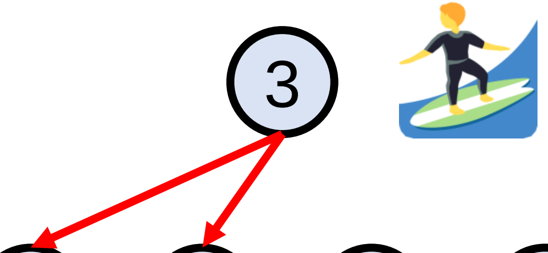
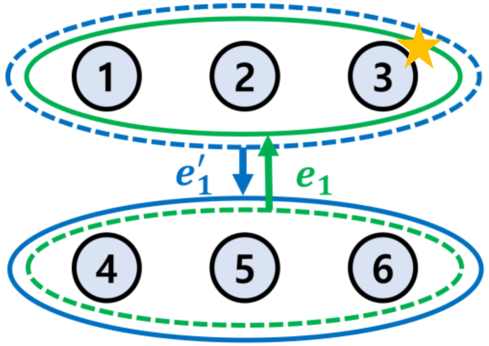
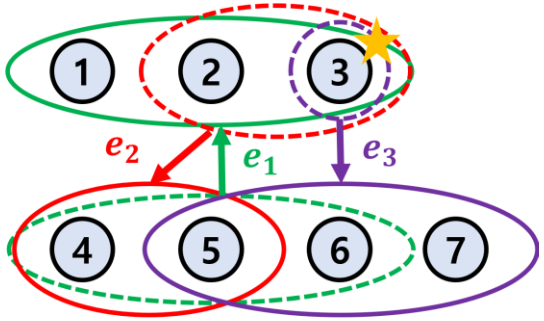


	1	2	3	4	5	6	7
Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
e_2	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0



Ideal



Observed

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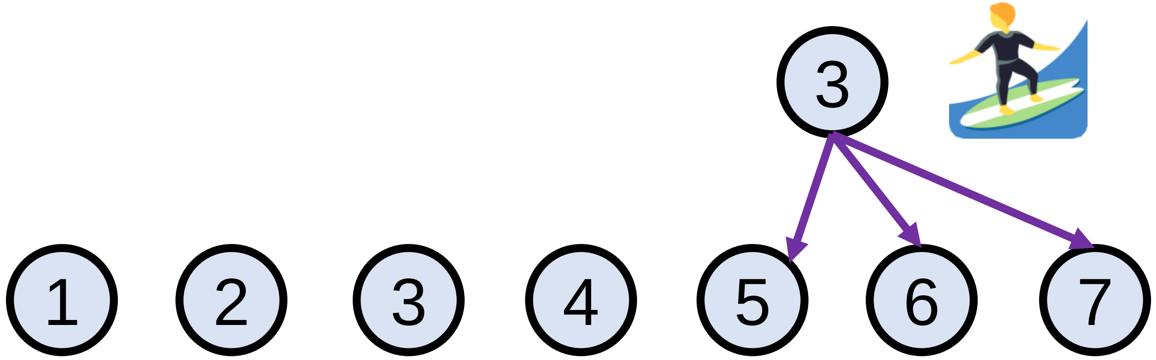
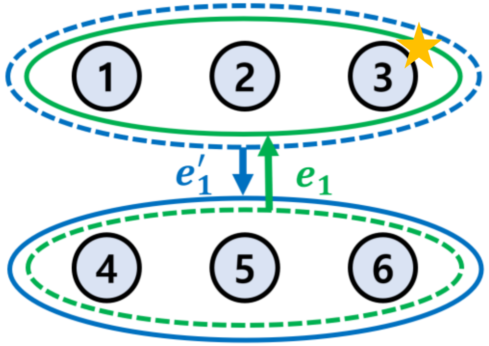


	1	2	3	4	5	6	7
Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
e_3	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$



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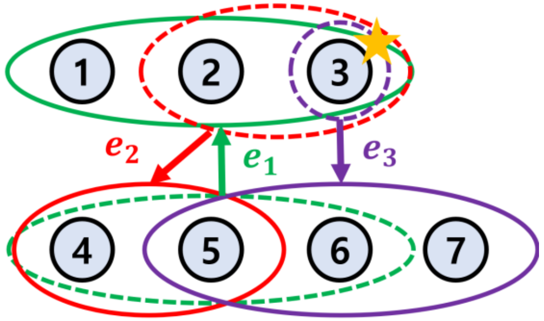

Observed

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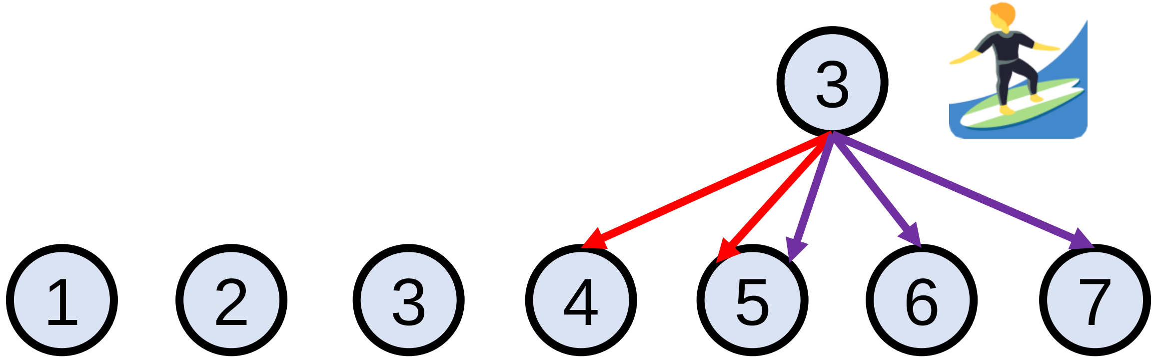
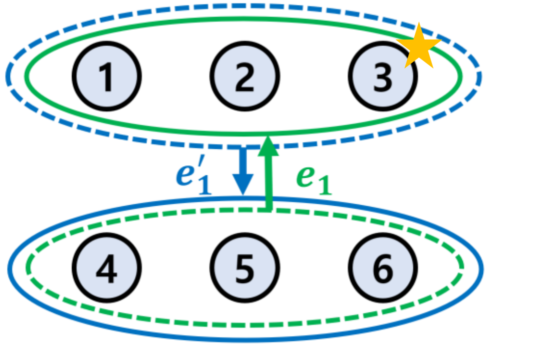


	1	2	3	4	5	6	7
Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
e_2	0	0	0	$\frac{1}{2}\cdot\frac{1}{2}$	$\frac{1}{2}\cdot\frac{1}{2}$	0	0
e_3	0	0	0	0	$\frac{1}{2}\cdot\frac{1}{3}$	$\frac{1}{2}\cdot\frac{1}{3}$	$\frac{1}{2}\cdot\frac{1}{3}$



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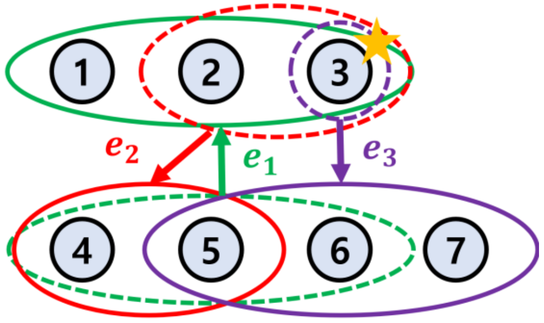

Observed

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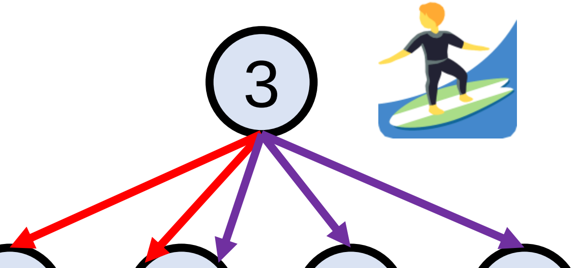
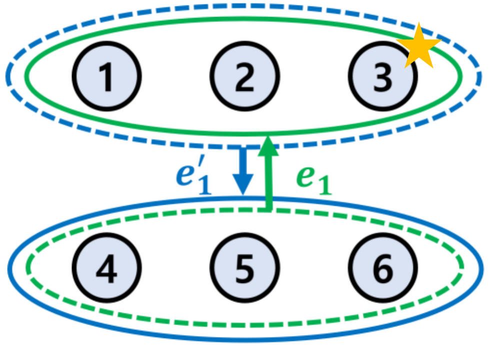
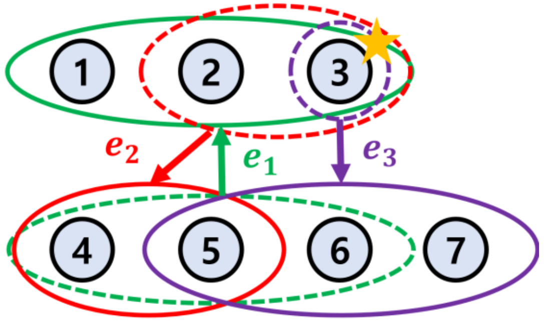


		1	2	3	4	5	6	7
Probability	Ideal	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0
Distance	e_2	0	0	0	$\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{2} \cdot \frac{1}{2}$	0	0
	+ e_3	0	0	0	0	$\frac{1}{2} \cdot \frac{1}{3}$	$\frac{1}{2} \cdot \frac{1}{3}$	$\frac{1}{2} \cdot \frac{1}{3}$



Ideal



Observed

HyperRec: Proposed Reciprocity Measure

- Hyperarc level HyperRec

$$r(e_i, R_i) = \left(\frac{1}{|R_i|}\right)^{\alpha} \left(1 - \frac{1}{|H_i|} \sum_{v \in \boxed{H_i}} \frac{\boxed{\mathcal{L}}(\boxed{p_v}, \boxed{p_v^*})}{\mathcal{L}_{max}}\right)$$

Head set of e_i

Probability distance measure (e.g., Jensen Shannon Divergence)

Optimal transition probability distribution

Transition probability distribution

HyperRec: Proposed Reciprocity Measure

Details

- Hyperarc level HyperRec

$$r(e_i) = \boxed{\max_{R_i \subseteq E, R_i \neq \emptyset}} \left(\frac{1}{|R_i|}\right)^{\boxed{\alpha}} \left(1 - \frac{1}{|H_i|} \sum_{v \in H_i} \frac{\mathcal{L}(p_v, p_v^*)}{\mathcal{L}_{max}}\right)$$

Reciprocal set size penalty term

Using a reciprocal set that can maximize the hyperarc reciprocity

- Hypergraph level HyperRec

$$r(G) = \frac{1}{\boxed{|E|}} \sum_{\boxed{e_i \in E}} r(e_i)$$

Average for every hyperarc

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Observation

- What are the real-world hypergraphs' reciprocal patterns?

Metabolic (iAF1260b & iJO1366)
- Node: Genes
- Hyperarcs: Reactions

Q&A (Math & Server)
- Node: Users
- Hyperarcs: Posts

Email (Enron & EU)
- Node: Users
- Hyperarcs: Mails

Citation (Data Mining & Software)
- Node: Authors
- Hyperarcs: Citations

Bitcoin (2014, 2015, 2016)
- Node: Users
- Hyperarcs: Transactions

Observation

Observation 1: Hypergraph Level.

Real-world hypergraphs are more reciprocal than randomized hypergraphs.

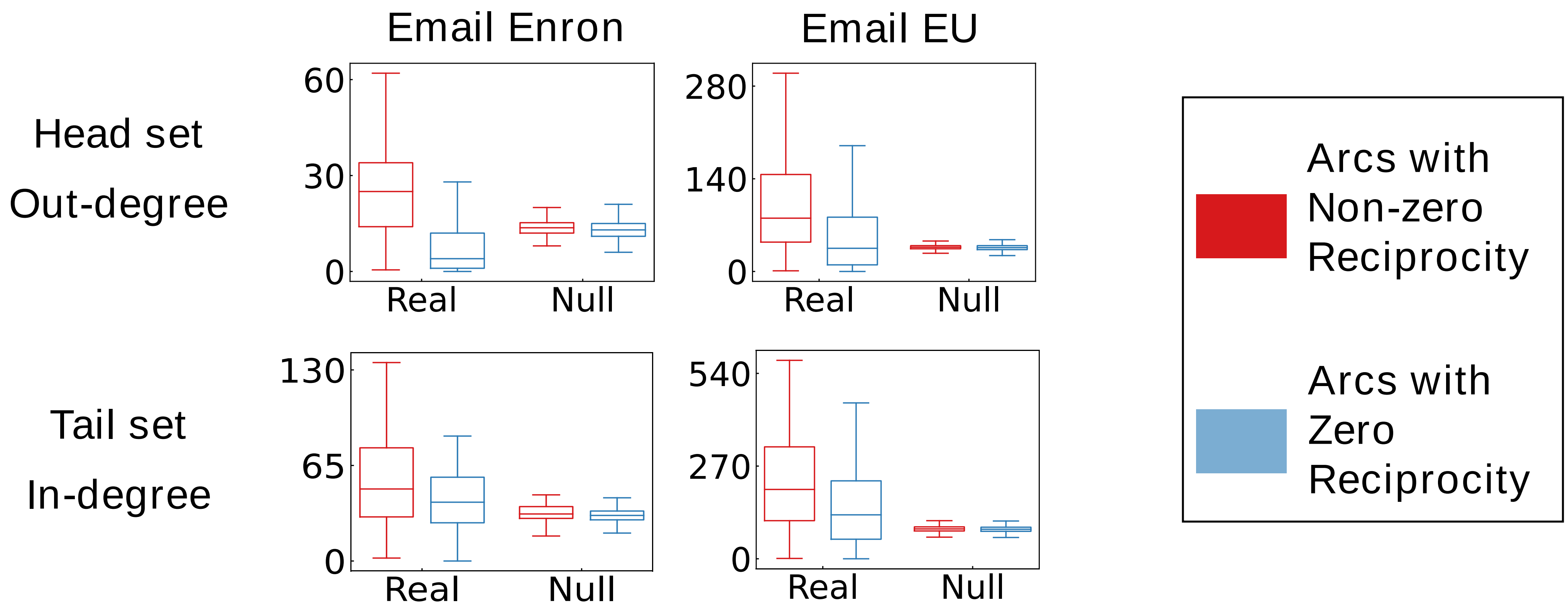
		metabolic		email		citation		q&a		bitcoin		
		iAF1260b	iJO1366	enron	eu	data mining	software	math	server	2014	2015	2016
Real World	$r(G)$	21.455	22.533	59.001	79.416	12.078	15.316	9.608	13.219	10.829	6.923	3.045
Null	$r(G)$	0.306	0.270	14.862	4.633	0.094	0.147	0.018	0.002	0.0001	0.000*	0.000*
	D-Stat	0.625	0.642	0.539	0.807	0.355	0.377	0.124	0.160	0.147	0.100	0.050

** Null hypergraph is a randomized hypergraph that fills nodes uniformly at random where hyperarc size is preserved.

Observation

Observation 2: Hyperarc Level.

Arcs with non-zero reciprocity tend to have higher head set out-degree and tail set in-degree.



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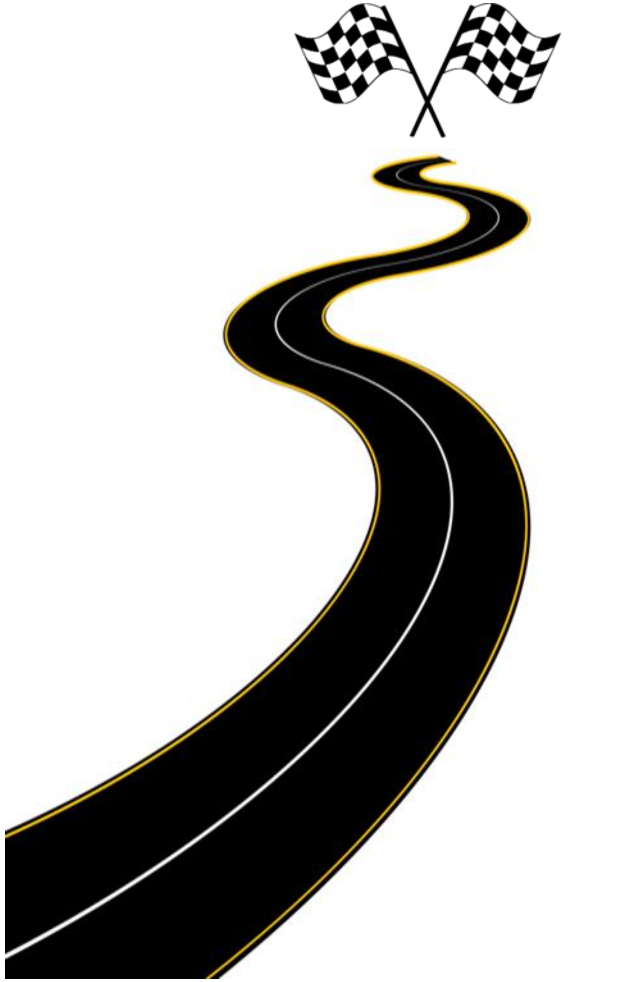
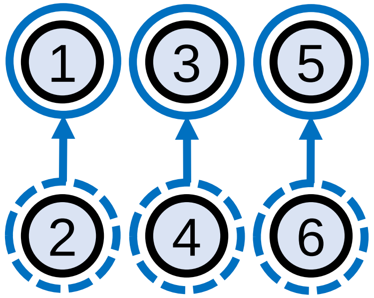


ReDi: Realistic Directed Hypergraph Generator

- Realistic ≒ Can preserve reciprocal patterns.
- ReDi iteratively add nodes and create hyperarcs ∝ group degree.
- Extent of reciprocity is being controlled by hyperparameters $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$.



0. Initialize with normal graph edges.
1. Add one node and decide the number of hyperarcs to be added with node 7.
2. Choose new hyperarcs' size.

ReDi: Realistic Directed Hypergraph Generator

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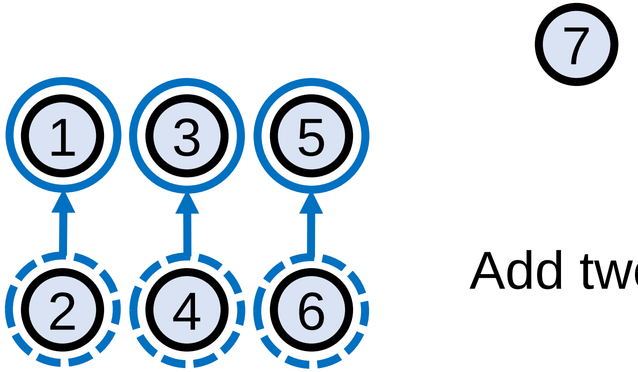

Add two hyperarcs!

0. Initialize with normal graph edges.
1. Add one node and decide the number of hyperarcs to be added with node 7.
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ReDi: Realistic Directed Hypergraph Generator

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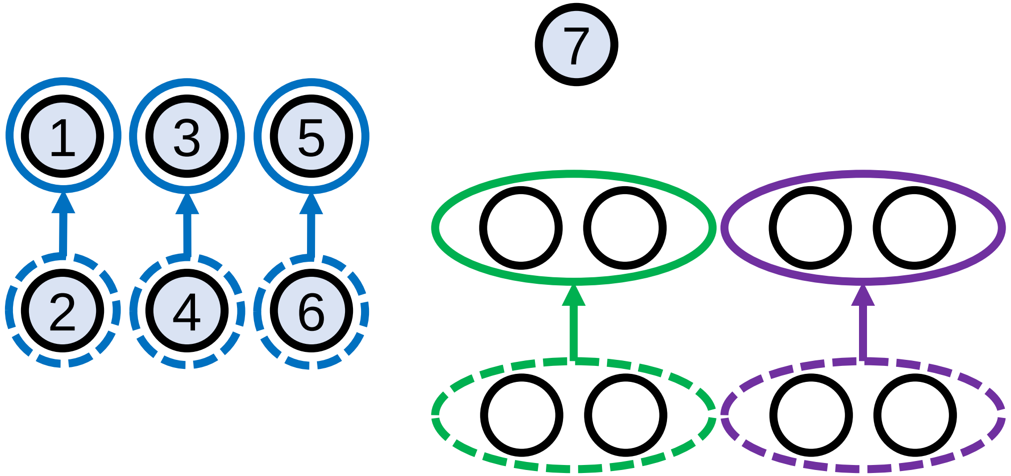

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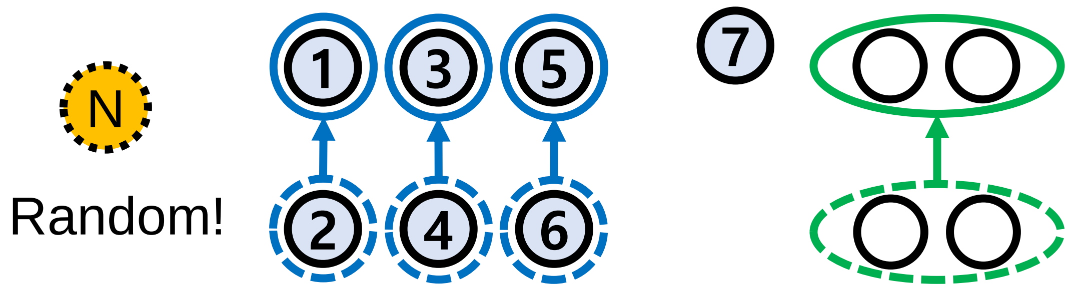

3. Toss a coin, and decide whether current hyperarc is reciprocal ($\propto \boldsymbol{\beta_1}$) or random ($\propto \mathbf{1} - \boldsymbol{\beta_1}$).
4. (Random) Put a new node in either head or tail set, and fill the remaining positions ∝ the group degree. Then add hyperarc.
5. (Reciprocal) Choose a hyperarc to be reciprocal.
6. Toss a coin ($\propto \boldsymbol{\beta_2}$), and decide how many and which nodes will we bring.
7. Fill remaining positions ∝ the group degree, and add hyperarc.

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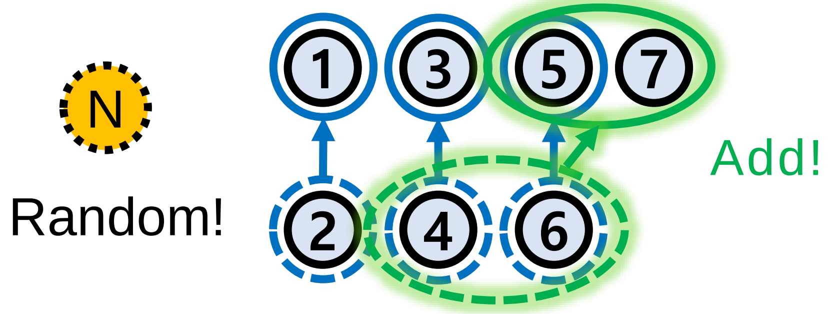

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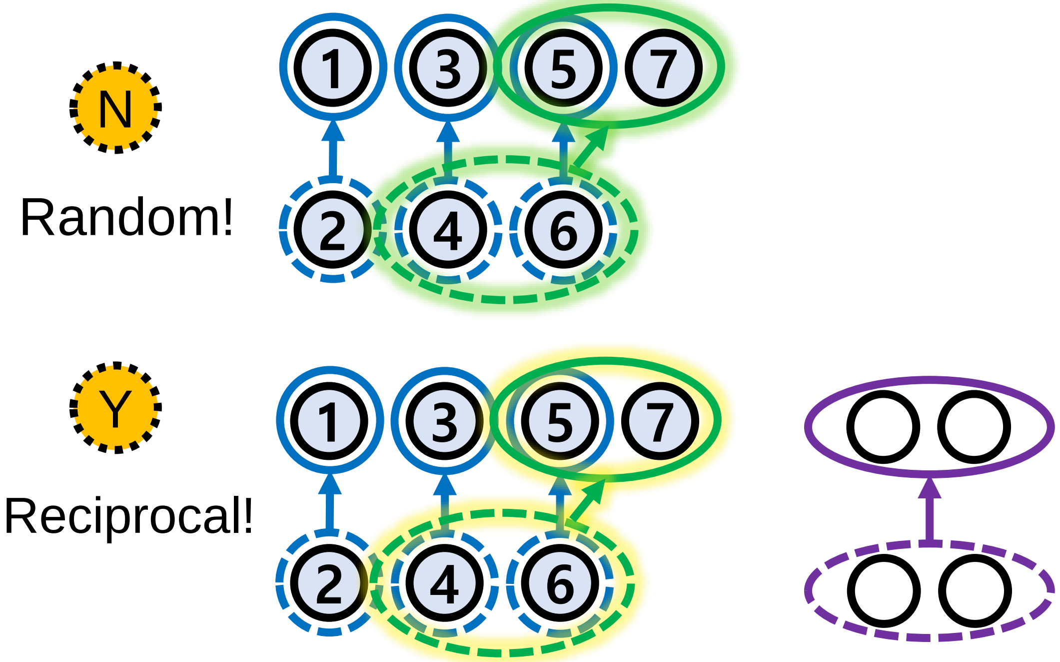
ReDi: Realistic Directed Hypergraph Generator



3. Toss a coin, and decide whether current hyperarc is reciprocal ($\propto \boldsymbol{\beta_1}$) or random ($\propto \mathbf{1} - \boldsymbol{\beta_1}$).
4. (Random) Put a new node in either head or tail set, and fill the remaining positions $\propto$ the group degree. Then add hyperarc.
5. (Reciprocal) Choose a hyperarc to be reciprocal.
6. Toss a coin ($\propto \boldsymbol{\beta_2}$), and decide how many and which nodes will we bring.
7. Fill remaining positions $\propto$ the group degree, and add hyperarc.

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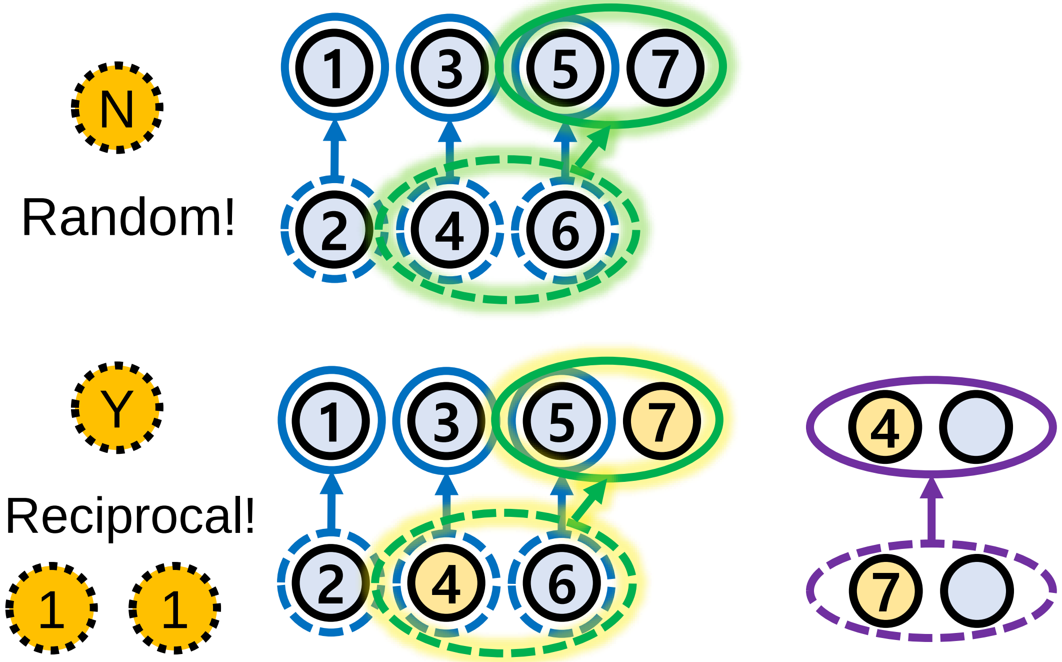

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4. (Random) Put a new node in either head or tail set, and fill the remaining positions ∝ the group degree. Then add hyperarc.
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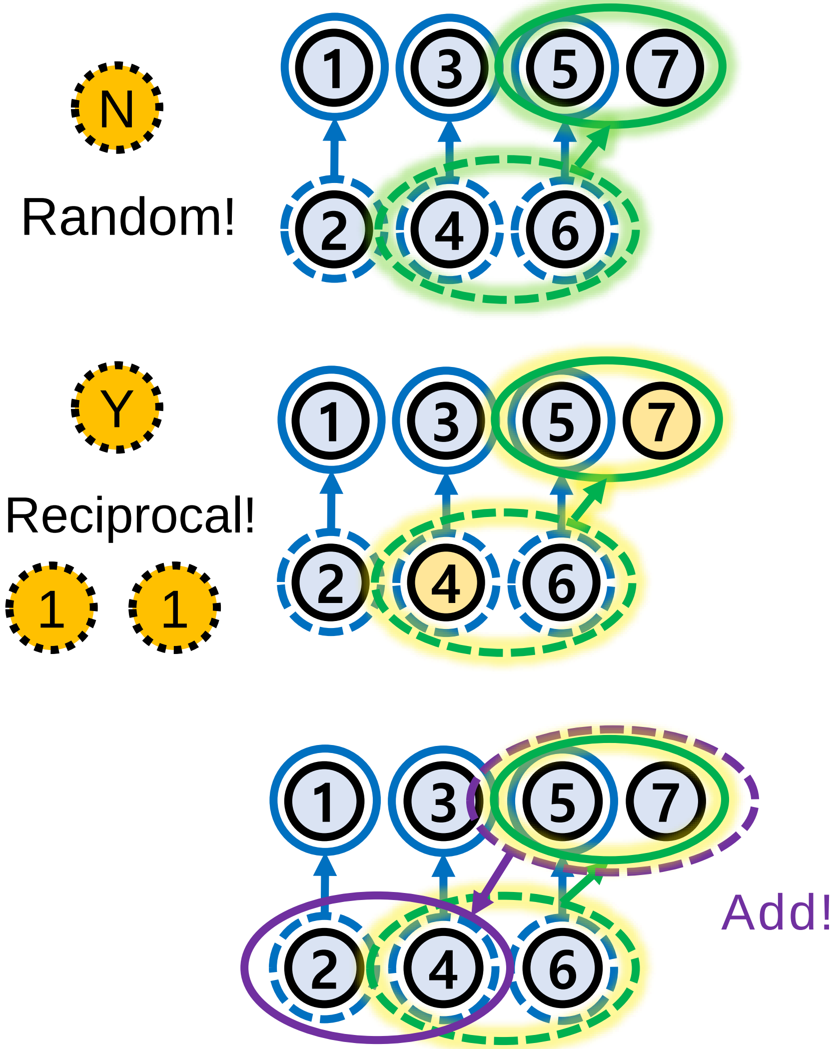

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Reproducibility of observation 1 of ReDi:

ReDi can preserve the reciprocity value of hypergraphs.

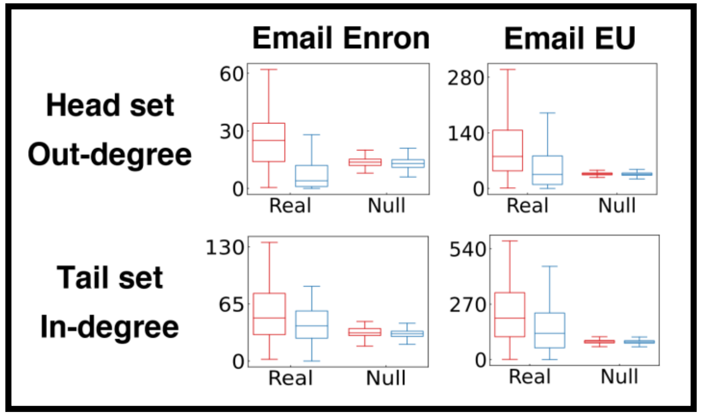
		metabolic		email		citation		q&a		bitcoin		
		iAF1260b	iJO1366	enron	eu	data mining	software	math	server	2014	2015	2016
Real World	$r(G)$	21.455	22.533	59.001	79.416	12.078	15.316	9.608	13.219	10.829	6.923	3.045
ReDi (Section 5)	$r(G)$	21.727	22.185	59.161	79.489	12.601	14.279	9.427	13.229	10.267	6.587	3.497
	D-Stat	0.098	0.104	0.053	0.043	0.212	0.151	0.011	0.005	0.045	0.033	0.017
Null	$r(G)$	0.306	0.270	14.862	4.633	0.094	0.147	0.018	0.002	0.0001	0.000*	0.000*
	D-Stat	0.625	0.642	0.539	0.807	0.355	0.377	0.124	0.160	0.147	0.100	0.050

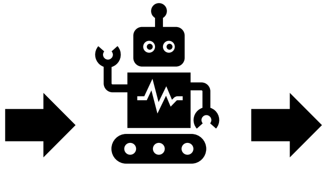
ReDi: Realistic Directed Hypergraph Generator

Reproducibility of observation 2 of ReDi:

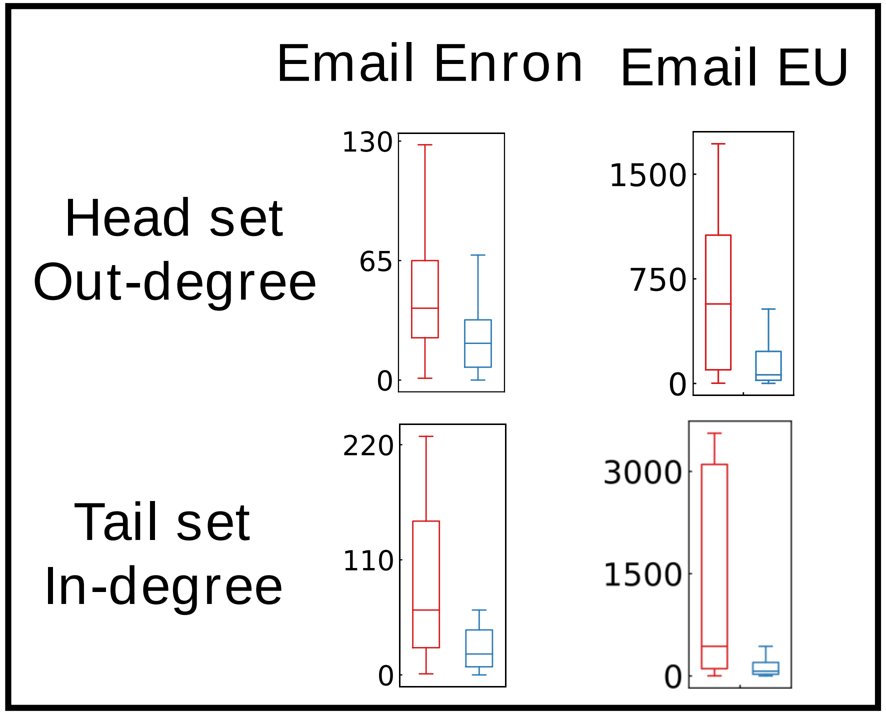
ReDi can preserve the relationship between arc degree and reciprocity.

Real-world





Generated



Arcs with Non-zero Reciprocity

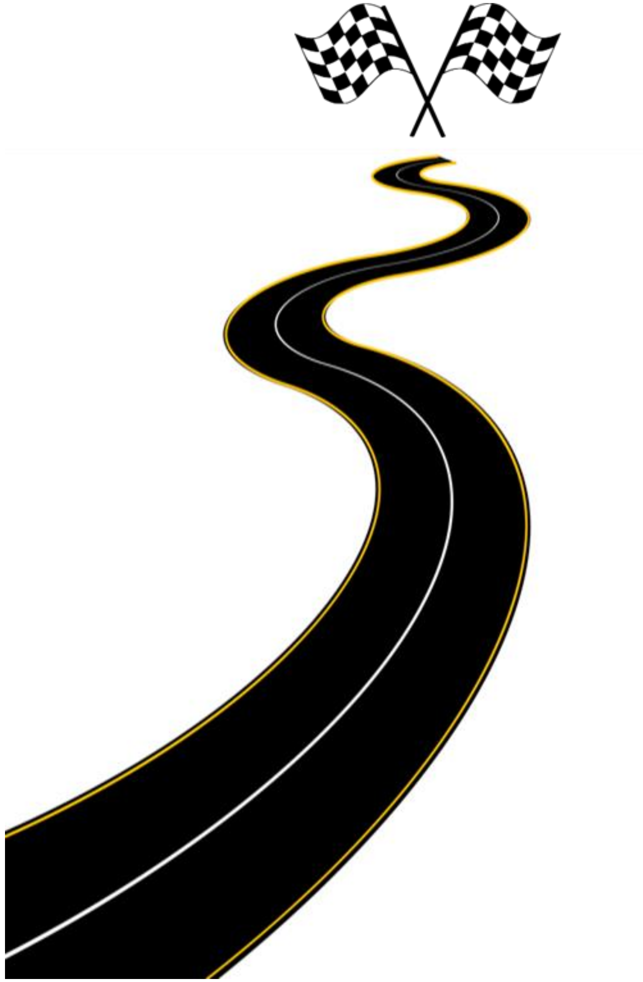
Arcs with Zero Reciprocity

Observation 2: Hyperarc Level.

Arcs with non-zero reciprocity tend to have higher head set out-degree and tail set in-degree.

Roadmap

- Overview
- Reciprocity Measure
- Observations
- Generators
- Conclusions



Conclusions

Our contributions in this work:

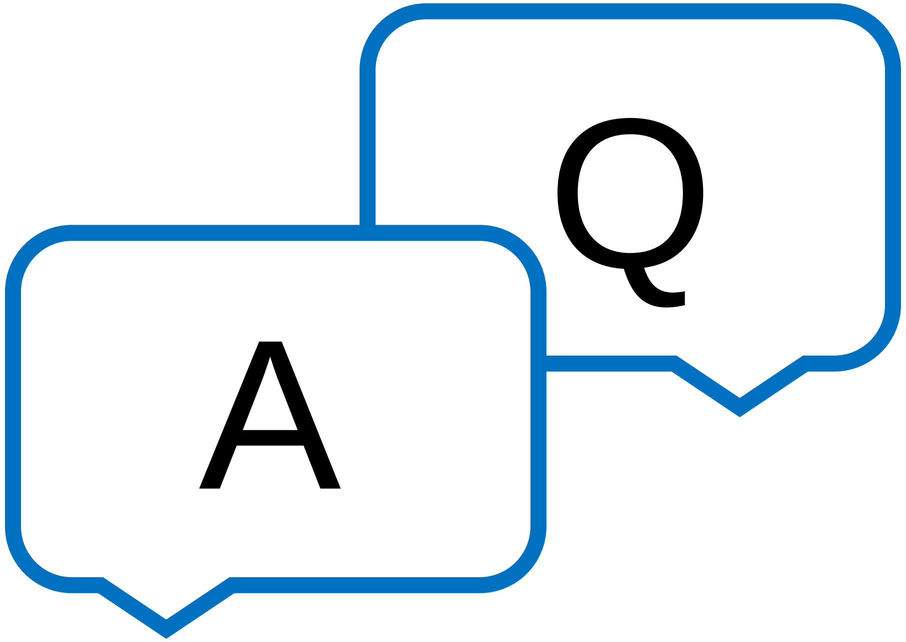
- Principled Reciprocal Measure: HyperRec
- Observations in Real-world Directed Hypergraphs
  1. Hypergraph level
  2. Hyperarc level
- Realistic Generative Model: ReDi

The code and datasets used in the paper are available at

https://github.com/kswoo97/hyprec

Skipped Details

- AXIOMs: Properties that a proper reciprocity measure should have.
- Reciprocal set: Composing a reciprocal set of the target arc.
- Components of HyperRec: Formal expression of HyperRec.
- Searching algorithm (Ferret): Finding a proper reciprocal set fast & accurately.
- Soundness of HyperRec: Theoretical analysis that a HyperRec can satisfy all the axioms.
- Exactness of Ferret: Theoretical analysis that a Ferret's output is accurate.
- Additional experimental results: Additional experiments and full dataset results regarding observations and generations.


Q
A