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In this document, we discuss future applications and directions of hypergraph mining, especially hypergraph patterns. We mainly review and discuss existing applications and research topics related to graph mining and graph patterns, especially the graph counterparts of what we have discussed in this survey. Since most hypergraph patterns are generalized from graph patterns, we expect many existing applications and directions of graph mining and graph patterns will also be extended and generalized to hypergraphs in the future.

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1 APPLICATIONS TO ALGORITHMIC DESIGN

First, we discuss the possible applications of hypergraph mining to algorithmic design. Many results of graph mining, especially graph patterns, have inspired the design of innovative graph algorithms for real-world applications. These algorithms have proved to be highly practical, demonstrating excellent efficiency and/or effectiveness. We expect that such applications can be generalized to hypergraphs in the future using the corresponding hypergraph patterns, and they can be useful for hypergraph algorithms and hypergraph mining, especially considering the high natural complexity of hypergraphs [42, 50, 77, 146].

Degree distributions and singular value distributions. The observation that real-world graphs usually exhibit heavy-tailed degree distributions has been used for the design of graph algorithms, including distributed graph algorithms [58, 137], degree distribution estimation algorithms [46], graph traversal algorithms [186], knowledge graph completion [145], and triangle counting algorithms [94]. Similarly, skewed singular values in real-world graphs have been used for optimizing triangle counting [90, 159]. Skewed degree distributions and singular value distributions are also observed in real-world hypergraphs (see P1 and P14). Therefore, the above applications are possibly extendable to hypergraphs, for the counterpart algorithmic problems on hypergraphs.

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Temporal locality. In many real-world temporal graphs, the temporal locality is observed, where edges appearing within a smaller temporal window are more likely to interact. This property has been used for designing efficient algorithms for triangle counting [100] and graph traversal [93]. Several patterns related to temporal locality have been observed in hypergraphs (see P16 and P20), and we expect such patterns to be useful in the design of algorithms for temporal hypergraphs.

Diameters. Small diameters in real-world graphs have been considered in designing algorithms for large-scale graph mining [86]. Therefore, shrinking diameters observed in real-world hypergraphs (see P26) are also possibly useful for large-scale hypergraph mining.

<u>Core-periphery structures</u>. Several algorithms leverage core-periphery structures in real-world graphs for efficient graph compression [109] and the rapid retrieval of similar nodes [82, 144], and thus we expect such structures in hypergraphs (see P3) to be useful in related tasks [35].

Treewidth. Bounded treewidth in real-world graphs has been used for optimizing graph queries [117], designing graph clustering algorithms [17], and Bayesian inference [140]. Although the study of treewidth in hypergraphs is still mainly limited to the theoretical field [114, 119, 156], we expect more patterns regarding treewidth will be discovered in real-world hypergraphs and those patterns will be used in many applications.

2 APPLICATIONS TO MACHINE LEARNING

In addition to algorithmic design, graph patterns have also been widely used in machine learning, especially machine learning on graphs. This suggests the potential usefulness of observed patterns within real-world hypergraphs across hypergraph-related applications, as discussed below.

Graph neural networks and general feature representation. One of the most common topics in machine learning on graphs is feature representation, where graph neural networks (GNNs) are often used. Many graph properties and patterns have been considered for enhancing the performance of GNNs, including degree distributions [107, 111, 174, 183], assortativity [155], graph motifs [18, 25, 41, 55, 80, 121, 128, 180, 189, 190], ego-networks [124, 138]. Structural patterns can also be used as additional node features to enrich the features used for graph learning [39, 64]. Recently, a line of research focused on using graph patterns for the theoretical analysis of GNNs. For example, graph motifs have been used to explain the learning process and the outcomes produced by GNNs [127], and ego-networks have been used for designing a theoretically and practically transferable GNN model [193]. Besides, graph patterns, especially graph motifs, can be used for general feature representation at both the node level and the graph level. Typically, graph motifs are extensively used for the comprehensive analysis and representation of whole graphs [11, 13, 120, 169], as well as for comparing multiple graphs [131, 175]. Moreover, graph motifs are also used for modeling the evolution of temporal graphs [38, 139]. We expect hypergraph patterns to be useful, as their graph counterparts, not only in GNNs (e.g., a straightforward generalization of k-cores in hypergraphs has been utilized for the initialization of GNNs [110]), but also in increasingly popular hypergraph neural networks [16, 29, 50, 56, 60, 66, 68, 76, 88, 108] and general feature representation in hypergraphs [12] for applications including educational management [105] and fake news detection [74].

Link prediction and community detection. Link prediction [96, 113] and community detection [53] are two traditional machine-learning problems on graphs. Many graph patterns have been used in those two problems, including assortativity [7, 36], graph motifs [1, 136, 164, 170], the structure of ego-networks [3, 157], and structural similarity (especially neighborhood homogeneity) [19, 27, 149, 166, 177]. Link prediction (i.e., hyperedge prediction) [26, 71, 97, 160, 163, 171, 173, 176] and community detection in hypergraphs [30, 37, 48, 84, 85, 87, 192] have gained more and more attention recently. Specific applications where they have been employed include (1) recognizing

unique sets of items to be purchased together [106], (2) proposing new combinations of ingredients for recipes [188], (3) suggesting novel collaborations among researchers [112], and (4) uncovering clusters of genes that collaborate for specific biological functions [122]. We look forward to seeing hypergraph patterns be used for these two tasks.

Anomaly detection. Anomaly detection [23] is another traditional machine-learning problem, and graph-based anomaly detection [5] is a popular subtopic. Many graph patterns have been used in graph-based anomaly detection, including graph motifs [123], the structure of ego-networks [4], k-cores [143], and structural similarity (especially neighborhood homogeneity) [22]. Recently, anomaly detection in hypergraphs has also been studied [101, 165], and Do and Shin [43] have considered a simple heuristic of anomaly detection on nodes by comparing the hypercoreness values and degrees of nodes. We anticipate more usage of hypergraph patterns for this application. Recommendation. Recommendation [73, 141] is a long-standing research topic in machine learning. Graphs are an important tool for building recommendation systems [24, 63, 153], and many graph patterns, including graph motifs [40, 59, 152, 191] and the structure of ego-networks [47], have been used in graph-based recommendation models. Furthermore, many challenges and remedies in recommendation systems are closely linked to graph patterns, such as addressing popularity bias stemming from heavy-tailed degree distributions [115, 168]. Hypergraphs are also useful for this task, especially bundle recommendation [154, 194] and group recommendation [9, 182], which can be modeled using hypergraphs [75, 118, 181, 185]. We await more applications of hypergraph patterns for recommendation systems.

<u>Subgraph sampling</u>. In machine learning on graphs, subgraph sampling is a useful technique for, e.g., better representation [8] and higher time efficiency [28, 142, 184]. A line of works exists on representative subgraph sampling [70, 103, 116], where the target is to sample subgraphs with similar characteristics as a given graph. Subgraph sampling has also been used for estimating quantities of a given graph [78, 179], where the estimation algorithms exploit the skewed degree distribution [2] and fast mixing time [61] of real-world graphs. Recently, this task has been considered on hypergraphs [34]. Hopefully, hypergraph patterns can be proven useful on related tasks, just like their graph counterparts.

3 ANALYSIS AND MINING OF GENERALIZED HYPERGRAPHS

In this survey, we have mainly discussed simple hypergraphs (i.e., undirected and unweighted ones). Below, we would like to discuss several types of generalized hypergraphs.

Directed hypergraphs. Directed hypergraphs, where nodes within each hyperedge are partitioned into a source set and a destination set, have been studied in the fields of theoretical mathematics and theoretical computer science with researchers paying continuous attention [14, 15, 54, 134]. Directed hypergraphs are applied to many tasks, including expert systems [132], image segmentation [44], music composition [67], metabolic network analysis [158], chemical reaction modeling [81], and objects retrieval [10]. Ranshous et al. [133] studied patterns in real-world directed transaction hypergraphs, and applied the observed patterns to transaction classification. Recently, Kim et al. [89] extended the concept of reciprocity to directed hypergraphs and studied related patterns within real-world directed hypergraphs. We expect more patterns to be explored on directed hypergraphs. **Weighted hypergraphs**. Most works mentioned in this survey deal with unweighted real-world

hypergraphs or explicitly preprocess the datasets into unweighted ones, although some take the repetition of hyperedges into consideration [20, 102]. At the same time, weighted hypergraphs provide a more general and expressive way to represent systems. Weighted hypergraphs, particularly those with each hyperedge associated with a numerical value, have been used for biological studies [72], image retrieval [69], concept-to-text generation [91, 92], and object classification [150].

Recently, there also has been a growing interest in hypergraphs with edge-dependent vertex weights (where a node can have different weights in different hyperedges) [31, 33, 62, 196]. We expect more patterns to be explored on weighted hypergraphs.

Heterogeneous hypergraphs. Heterogeneous hypergraphs are another type of generalized hypergraphs, where nodes can belong to different classes (or types, labels, etc.). Some theoretical studies have been conducted on heterogeneous hypergraphs [148, 162]. Recently, heterogeneous hypergraphs have also been considered for hypergraph representation learning [49, 151, 172, 195]. More patterns await discovery on heterogeneous hypergraphs.

<u>Uncertain hypergraphs</u>. Generalized hypergraphs also include uncertain hypergraphs, where the presence or absence of hyperedges is not deterministic but governed by probabilities or uncertainty measures. Uncertainty naturally arises in real-world scenarios, and it is important to consider uncertainty when modeling real-world systems into graphs or hypergraphs [21, 79, 129]. The studies on uncertain hypergraphs are still mainly limited to theoretical ones [126, 147, 167, 187], and we expect that more patterns can be discovered on uncertain hypergraphs.

4 OUT-OF-SCOPE DISCUSSIONS

In this survey, we have focused on tools, measures, and generators that are used for or based on patterns in real-world hypergraphs. Below, we would like to provide discussions on related work that is out of the scope of this survey.

Distances. Recently, some researchers have tried to propose distance metrics in hyperedges. Vasilyeva et al. [161] proposed distance metrics based on the random walks in weighted line graphs of hypergraphs. Aksoy et al. [6] proposed a distance metric considering higher-order connectivity in hypergraphs, and Preti et al. [130] proposed a fast approximation algorithm of such a metric. Li and Fadlallah [104] considered another distance metric based on the expected hitting times of random walks in hypergraphs, and they proposed an efficient computational algorithm for the metric. The above distance measures have not been used for mining real-world patterns yet.

<u>Mathematical models.</u> There are also various existing mathematical models of hypergraphs [32, 37, 45, 51, 52, 57, 65, 83, 95, 98, 99, 125, 135, 178], where real-world hypergraph properties and patterns are not considered. Using the mathematical ideas and tools in such works to enhance hypergraph generation might be an interesting future direction.

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